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Asymptotic behavior of solutions of perturbed linear difference systems ☆

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Abstract

This paper is concerned with asymptotic behavior of solutions of perturbed linear difference systems. Several asymptotic results are obtained, some of which can be regarded as discrete analogs of the famous asymptotic results for differential systems, including the Hartman–Wintner theorem, the Harris–Lutz theorem, and the Eastham theorem. In addition, the conditions of a result established by Z. Benzaid and D.A. Lutz are weakened.

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1. Introduction

Consider the following perturbed linear diagonal difference system

$$y(t+1) = (\Lambda(t) + R(t))y(t), \quad t \in [t_0, +\infty), \tag{1.1}$$

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and the following perturbed linear constant difference system

$$y(t+1) = (C + R(t))y(t), \quad t \in [t_0, +\infty), \tag{1.2}$$

where $\Lambda(t) = \operatorname{diag}(\lambda_1(t), \ldots, \lambda_k(t))$, C, and $R(t)$ are $k \times k$ real or complex matrices; C is a constant matrix; $R(t)$ is a small perturbation in some sense; and the interval $[t_0, +\infty) := \{t\}_{t=t_0}^{+\infty}$. In this paper, we always assume that $\Lambda(t) + R(t)$ and $C + R(t)$ are invertible on the interval $[t_0, +\infty)$.

In 1948, Levinson studied asymptotic behavior of solutions of the perturbed differential system

$$y'(x) = (\Lambda(x) + R(x))y(x),$$

where $\Lambda(x)$ is a diagonal matrix, and established an important asymptotic result, called the Levinson Theorem (see [13] or [5–Theorem 8.1 in Chapter 3] or [7–Theorem 1.3.1]), which played an important role in the study of asymptotic problems of perturbed differential systems. Hartman and Wintner [11] got another important result, called the Hartman–Wintner theorem, in 1955. Later, their works were followed by Harris and Lutz [9,10], Eastham [7], etc. Many excellent asymptotic results for differential systems were summarized in the monograph of Eastham [7], and many references were cited therein.

In the existing literature on research of perturbed linear difference systems, in 1911 Birkhoff [2] studied asymptotic behavior of solutions of system (1.2) in which $R(t)$ has a convergent or asymptotic power series in t^{-1} for t in some open interval containing the positive real axis. Coffman [6] considered asymptotic behavior of solutions of difference equations with almost constant coefficients. Later, Benzaid and Lutz [1] got several asymptotic results, one of which is a discrete analog of the Levinson theorem, which plays an important role in our paper. More recently, Bohner and coworkers [3,4] investigated asymptotic behavior of dynamic equations on time scales.

In this paper, similarly to the case of differential systems, two types of conditions are crucial in studying asymptotic representations of solutions: the first is a dichotomy condition on the diagonal matrix $\Lambda(t)$, and the second is a growth condition on the perturbation term $R(t)$. These two conditions are interrelated, and so we can obtain asymptotic representations of solutions in variety of ways by strengthening one condition while weakening the other one. In this paper, we establish several asymptotic results that can be regarded as discrete analogs of the well-known Hartman–Wintner theorem [7–Theorem 1.5.1], the Harris–Lutz theorem [7–Theorem 1.5.2], and the Eastham theorem [7–Theorem 1.6.1]. In 1987, Benzaid and Lutz established a discrete analog of the Hartman–Wintner theorem (see [1–Corollary 3.4]). However, we shall remark that our results can not be included by the existing results and especially the conditions of our discrete analog of the Hartman–Wintner theorem (i.e. Theorem 3.1 in Section 3) are weaker than those of [1–Corollary 3.4] (see Remark 3.2). Applications of these results to deficiency index and spectrum of the difference operators will be discussed in our forthcoming papers.