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Layout of random circulant graphs

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A circulant graph G is a graph on n vertices that can be numbered from 0 to $n-1$ in such a way that, if two vertices x and $(x+d)$ mod n are adjacent, then every two vertices z and $(z+d)$ mod n are adjacent. We call layout of the circulant graph any numbering that witness this definition. A random circulant graph results from deleting each edge of G uniformly with probability $1-p$. We address the problem of finding the layout of a random circulant graph. We provide a polynomial time algorithm that approximates the solution and we bound the error of the approximation with high probability.

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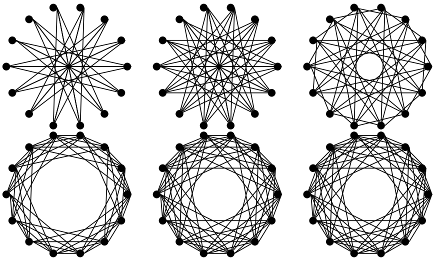

Fig. 1.1. Circulant graphs.

1. Introduction

A layout on the graph $G = (V, E)$ is a bijection function $f : V \to \{1, \ldots, |V|\}$. Layout problems can be used to formulate several well-known optimization problems on graphs. Also known as linear ordering problems or linear arrangement problems, they consist on the minimization of specific metrics. Such metrics would provide the solution to problems as linear arrangement, bandwidth, modified cut, cut width, sum cut, vertex separation and edge separation. All these problems are NP-hard in the general case.

A circulant graph H is defined on the set of vertices $V = \{1, \ldots, n\}$ and edges $E = \{(i,j) : |i-j| \equiv s\,(\text{mod} n)\,, s \in N\}$, where $N \subseteq \left\{1, \ldots, \lceil \frac{n-1}{2} \rceil\right\}$. There are a few equivalent definitions for a circulant graph: these graphs have a circulant adjacency matrix; a circulant graph H is a graph on n vertices that can be numbered from 0 to $n-1$ in such a way that, if two vertices x and $(x+d)$ mod n are adjacent, then every two vertices z and $(z+d)$ mod n are adjacent. We call layout of the circulant graph any numbering that witness the latter definition. A random circulant graph results from deleting each edge of H uniformly with probability $1-p$. Finally, a layout of the random circulant graph is a layout of the deterministic graph H, which we refer as the graph model. Noticeable, circulant graphs carry a nice shape (see Fig. 1.1).

From the picture, it is easy to see that starting from zero any sequential numbering given to the vertices around the circle is a layout. In fact, any layout arises this way if we correctly place all vertices around a circle. That means a layout encodes the geometric structure of a circulant graph. Naturally, the structure is expected to be reflected in the random graph as well, which gives rise to the main question of this paper. Precisely, the problem we address is the following: given a random circulant graph, find its layout. In this paper, we present a solution to this problem by means of eigenvectors.

This type of problems is related to the Minimum Linear Arrangement problem (MinLA), that is, to find a function f that minimizes the sum $\sum_{uv \in E} |f(u) - f(v)|$. The MinLA is one of the most important graph layout problems and was introduced in 1964 by Harper to develop error-correcting codes with minimal average absolute errors. In fact, MinLA appear in a vast domain of problems: VLSI circuit design, network reliability, topology awareness of overlay networks, single machine job scheduling, numerical analysis, computational biology, information retrieval, automatic graph drawing, etc. For instance, layout problems appear in the reconstruction of DNA sequences [6],