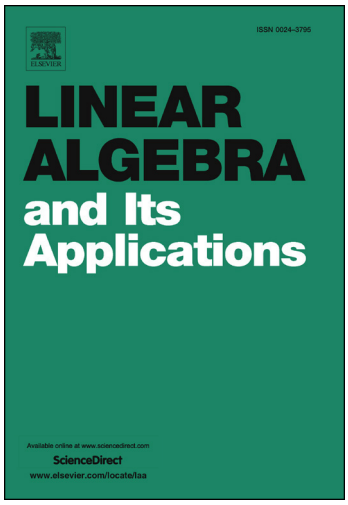

Explicit Solutions of Infinite Systems of Linear Equations from Reflexive Generalized Inverses of finite potent endomorphisms

Víctor Cabezas Sánchez, Fernando Pablos Romo

PII: S0024-3795(18)30431-2
DOI: https://doi.org/10.1016/j.laa.2018.09.004
Reference: LAA 14719

To appear in: *Linear Algebra and its Applications*

Received date: 2 May 2018
Accepted date: 4 September 2018

Please cite this article in press as: V. Cabezas Sánchez, F. Pablos Romo, Explicit Solutions of Infinite Systems of Linear Equations from Reflexive Generalized Inverses of finite potent endomorphisms, *Linear Algebra Appl.* (2018), https://doi.org/10.1016/j.laa.2018.09.004

EXPLICIT SOLUTIONS OF INFINITE SYSTEMS OF LINEAR EQUATIONS FROM REFLEXIVE GENERALIZED INVERSES OF FINITE POTENT ENDOMORPHISMS

VÍCTOR CABEZAS SÁNCHEZ (*)
FERNANDO PABLOS ROMO (**)

Abstract. The aim of this work is to offer a method for computing reflexive generalized inverses of finite potent endomorphisms, that can be applied to obtain explicit solutions of infinite systems of linear equations.

Contents

1. Introduction

Let k be a fixed ground field and V a vector space over k. If we consider an endomorphism φ of V, according to [5] it can be said that φ is "finite-potent" if $\varphi^n V$ is finite dimensional for some n.

The definition of a reflexive generalized inverse of a singular $n \times n$-matrix is well-known. From this definition, reflexive generalized inverses of endomorphisms of finite-dimensional vector spaces make sense. Given an arbitrary k-vector space, the aim of this work is to offer a method for computing reflexive generalized inverses of a finite potent endomorphism φ. Thus, for each Jordan basis B of V determined by φ, we shall construct a reflexive generalized inverse φ_B^+ of φ. The linear map $\varphi_B^+ \in \mathrm{End}_k(V)$ depends on the choice of the basis B, is also finite potent, and is equivalent to φ under the action of the group of automorphisms of V.

As an application of this construction we shall obtain explicit solutions of infinite systems of linear equations. For an index family I, we shall compute the solutions on $\prod_{i\in I} k$ of a system of linear equations

$$\{\sum_{i\in I} a_{ij}x_j = \alpha_i\}_{i\in I}\,, \tag{1.1}$$

Key words and phrases. vector space, generalized inverse, finite potent endomorphism, infinite linear system.

2010 Mathematics Subject Classification: 15A09, 15A06, 15A04

This work is partially supported by a collaboration-fellowship provided by the Spanish Government (*), and by the Spanish Government research contract no. MTM2015-66760-P and by the Castilla y León Regional Government research contract no. SA030G18 (**).