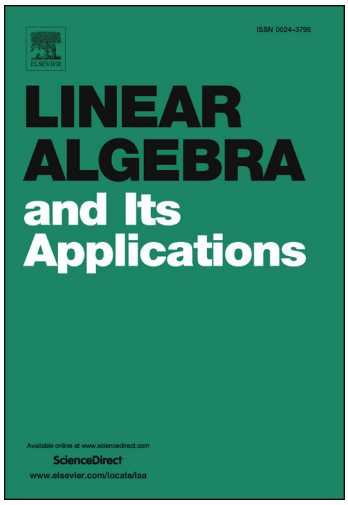

Perturbed marked reduced forms of invariant subspaces

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Perturbed Marked Reduced forms of invariant subspaces *

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Abstract

The classification of invariant subspaces is an open problem related to other important ones like the Carlson problem. Here we obtain a reduced form of these invariant subspaces as a new tool to tackle these problems. In particular, it allows us to prove quite easily partial results already known. The key point is assigning to each invariant subspace a marked one (its marked type) in order to partition the set of invariant subspaces in a finite number of subsets (the marked classes), each one containing only one marked subspace. Next, we parametrize (minimally) each marked class by means of the so-called PM reduced families, so that representatives of an invariant subspace (its PM reduced forms) appear in just one of these families.

AMS Mathematics Subject Classification : 15A21, 93B10, 15A03.

Keywords: endomorphism, invariant subspaces, marked subspaces.

1 Introduction

Given an N-dimensional $\mathbb{C}$-vector space E, we consider the study of pairs $(\mathcal{V}, f)$, where f runs over $End(E)$ and, for each one, $\mathcal{V}$ runs over the f-invariant subspaces. We recall that for an endomorphism $f \in End(E)$, a subspace $\mathcal{V} \subset E$ is called *f-invariant* if $f(\mathcal{V}) \subset \mathcal{V}$, and two invariant pairs $(\mathcal{V}, f)$ and $(\mathcal{V}', f')$ are called *equivalent* if there is an automorphism φ such that $\varphi(\mathcal{V}) = \mathcal{V}'$ and $\varphi \circ f = f' \circ \varphi$. The classification of invariant pairs according to this equivalence relation is an open problem, with partial results for quite particular cases: [11] and [4] for monogenic subspaces; in [12] one proves that there is a finite number of equivalence classes if the degree of the minimal polynomial prime factors is less than or equal to 5, and one asserts that the classification problem is "wild" when this degree is greater than 6.

We can restrict ourselves (see section 2.1) to f being a nilpotent endomorphism with a prefixed Segre characteristic $p = (p_1, ..., p_n)$, $\mathcal{V}$ being a d-dimensional f-invariant subspace and the restriction of f to $\mathcal{V}$ having a prefixed Segre characteristic $q = (q_1, ..., q_m)$. We denote by $Inv(p, q)$

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