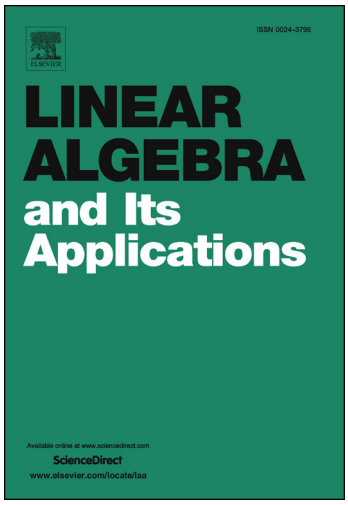

Characterization of half-radial matrices

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Characterization of half-radial matrices

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Abstract

Numerical radius $r(A)$ is the radius of the smallest ball with the center at zero containing the field of values of a given square matrix A. It is well known that $r(A) \leq \|A\| \leq 2r(A)$, where $\|\cdot\|$ is the matrix 2-norm. Matrices attaining the lower bound are called radial, and have been analyzed thoroughly. This is not the case for matrices attaining the upper bound where only partial results are available. In this paper we consider matrices satisfying $r(A) = \|A\|/2$, and call them half-radial. We summarize the existing results and formulate new ones. In particular, we investigate their singular value decomposition and algebraic structure, and provide other necessary and sufficient conditions for a matrix to be half-radial. Based on that, we study the extreme case of the attainable constant 2 in Crouzeix's conjecture. The presented results support the conjecture of Greenbaum and Overton, that the Crabb-Choi-Crouzeix matrix always plays an important role in this extreme case.

Keywords: field of values, numerical radius, singular subspaces, Crouzeix's conjecture
2000 MSC: 15A60, 47A12, 65F35

1. Introduction

Consider the space $\mathbb{C}^n$ endowed with the Euclidean vector norm $\|z\| = (z^*z)^{1/2}$ and the Euclidean inner product $\langle z, w\rangle = w^*z$, where $*$ denotes the Hermitian transpose. Let $A \in \mathbb{C}^{n\times n}$. The *field of values* (or numerical range)

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