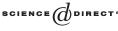


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## The rate of convergence of *q*-Bernstein polynomials for 0 < q < 1

Heping Wang\*, Fanjun Meng

Department of Mathematics, Capital Normal University, Beijing 100037, People's Republic of China

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## Abstract

In the note, we obtain the estimates for the rate of convergence for a sequence of *q*-Bernstein polynomials  $\{B_{n,q}(f)\}$  for 0 < q < 1 by the modulus of continuity of *f*, and the estimates are sharp with respect to the order for Lipschitz continuous functions. We also get the exact orders of convergence for a family of functions  $f(x) = x^{\alpha}$ ,  $\alpha > 0$ ,  $\alpha \neq 1$ , and the orders do not depend on  $\alpha$ , unlike the classical case.

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## 1. Introduction

Let q > 0. For each nonnegative integer k, the q-integer [k] and the q-factorial [k]! are defined by

$$[k] := \begin{cases} (1-q^k)/(1-q), & q \neq 1 \\ k, & q = 1 \end{cases}$$

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<sup>\*</sup> Corresponding author.

E-mail address: wanghp@mail.cnu.edu.cn (H. Wang).

and

$$[k]! := \begin{cases} [k] [k-1] \cdots [1], & k \ge 1\\ 1, & k = 0 \end{cases}$$

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respectively. For the integers n, k,  $n \ge k \ge 0$ , the q-binomial coefficients are defined by (see [3, p. 12])

$$\begin{bmatrix} n\\k \end{bmatrix} := \frac{[n]!}{[k]![n-k]!}$$

In 1997, Phillips proposed the following *q*-Bernstein polynomials  $B_{n,q}(f, x)$ . For each positive integer *n*, and  $f \in C[0, 1]$ , we define

$$B_{n,q}(f,x) := \sum_{k=0}^{n} f\left(\frac{[k]}{[n]}\right) {n \brack k} x^{k} \prod_{s=0}^{n-k-1} (1-q^{s}x),$$
(1.1)

where it is agreed that an empty product denotes 1 (see [6]). When q = 1,  $B_{n,q}(f, x)$  reduce to the well-known Bernstein polynomials  $B_n(f, x)$ :

$$B_n(f,x) := \sum_{k=0}^n f\left(\frac{k}{n}\right) \binom{n}{k} x^k (1-x)^{n-k}.$$

In recent years, the *q*-Bernstein polynomials have attracted much interest, and a great number of interesting results related to the *q*-Bernstein polynomials have been obtained (see [2,5–9]). This note is concerned with the quantitative results for the rate of convergence of the *q*-Bernstein polynomials for 0 < q < 1. For  $f \in C[0, 1]$ , t > 0, we define the modulus of continuity  $\omega(f, t)$  and the second modulus of smoothness  $\omega_2(f, t)$  as follows:

$$\omega(f;t) := \sup_{\substack{|x-y| \le t \\ x,y \in [0,1]}} |f(x) - f(y)|,$$
  
$$\omega_2(f,t) := \sup_{0 < h \le t} \sup_{x \in [0,1-2h]} |f(x+2h) - 2f(x+h) + f(x)|$$

For fixed  $q \in (0, 1)$ , II'inskii and Ostrovska proved in [2] that for each  $f \in C[0, 1]$ , the sequence  $\{B_{n,q}(f, x)\}$  converges to  $B_{\infty,q}(f, x)$  as  $n \to \infty$  uniformly for  $x \in [0, 1]$ , where

$$B_{\infty,q}(f,x) := \begin{cases} \sum_{k=0}^{\infty} f(1-q^k) \frac{x^k}{(1-q)^k [k]!} \prod_{s=0}^{\infty} (1-q^s x), & 0 \le x < 1, \\ f(1), & x = 1. \end{cases}$$
(1.2)

The first author of the note gave the following quantitative result for the rate of convergence of the *q*-Bernstein polynomials (see [9]):

$$\|B_{n,q}(f) - B_{\infty,q}(f)\| \leq c \,\omega_2(f,\sqrt{q^n})$$

$$\tag{1.3}$$

with  $\|\cdot\|$  the uniform norm, here *c* is an absolute constant. Note that when  $f(x) = x^2$ , we have (see [9]):

$$\|B_{n,q}(f) - B_{\infty,q}(f)\| = \sup_{x \in [0,1]} \frac{q^n (1-q)}{1-q^n} x(1-x) \asymp q^n \asymp \omega_2(f, \sqrt{q^n}),$$

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