Available online at www.sciencedirect.com
SCIENCE DIRECT®

J. Differential Equations 218 (2005) 69–90

Journal of Differential Equations

www.elsevier.com/locate/jde

Singular perturbations for third-order nonlinear multi-point boundary value problem☆

Zengji Du[a,*], Weigao Ge[a], Mingru Zhou[b]

[a]Department of Mathematics, Beijing Institute of Technology, Beijing 100081, PR China
[b]Department of Mathematics, Xuzhou Normal University, Xuzhou, Jiangsu, 221116, PR China

Received 9 July 2004; revised 6 January 2005

Available online 2 March 2005

Abstract

This paper is devoted to study the following third-order multi-point singularly perturbed boundary value problem

$$\varepsilon x'''(t) + f(t, x(t), x'(t), x''(t), \varepsilon) = 0, \quad 0 \leqslant t \leqslant 1, \ \ 0 < \varepsilon \ll 1,$$

$$x(0, \varepsilon) = 0,$$

$$ax'(0, \varepsilon) - bx''(0, \varepsilon) + \textstyle\sum_{i=1}^{n-2} \alpha_i x(\xi_i, \varepsilon) = A,$$

$$cx'(1, \varepsilon) + dx''(1, \varepsilon) + \textstyle\sum_{i=1}^{n-2} \beta_i x(\eta_i, \varepsilon) = B,$$

where $a, b, c, d \geqslant 0$, $A, B \in R$, $a+b>0$, $c+d>0$, $\alpha_i \leqslant 0$, $\beta_i \leqslant 0$, $i = 1, 2, \ldots, n-2$, $0 < \xi_1 < \xi_2 < \cdots < \xi_{n-2} < 1$, and $0 < \eta_1 < \eta_2 < \cdots < \eta_{n-2} < 1$. The existence, uniqueness and asymptotic estimates of solutions of the boundary value problem are give by using priori estimates, differential inequalities technique and Leray–Schauder degree theory.

Keywords: Singular perturbations; Nonlinear boundary value problem; Existence and uniqueness; Asymptotic estimates; Upper and lower solutions; Leray–Schauder degree

☆ Sponsored by the National Natural Science Foundation of China (No. 10371006).
* Corresponding author.
E-mail addresses: duzengji@163.com (Z. Du), gew@bit.edu.cn (W. Ge), zhoumr@xznu.edu.cn (M. Zhou).

0022-0396/$ - see front matter © 2005 Elsevier Inc. All rights reserved.
doi:10.1016/j.jde.2005.01.005

1. Introduction

For the past 20 years an extensive research has been made on the existence and asymptotic estimates of singularly perturbed boundary value problem for third-order nonlinear differential equation, for instance, to see [5,9,12–15] and therein. Many techniques arose in the studies of this kind of problem. For example, Howes [9] has considered problems of type

$$\varepsilon^2 y''' = f(y)y' + g(x, y),\ y(a) = A,\ y(b) = B,\ y'(b) = C$$

and discussed the existence and asymptotic estimates of the solutions by the method of descent. Zhao [15] has discussed a more general class of third-order singularly perturbed boundary value problems of the form

$$\varepsilon y''' = f(x, y, y', \varepsilon),\ y'(0) = 0,\ y(1) = 0,\ y'(1) = 0$$

and discussed the existence of solution and obtained asymptotic estimates using the theory of differential inequalities. Feckan [5] has studied high-order problems and his approach was based on the nonlinear analysis involving fixed-point theory, Leray–Schauder theory, etc. Valarmathi and Ramanujam [13] has considered singularly perturbed third-order ordinary differential equations of convection–diffusion type

$$\begin{aligned} &-\varepsilon y'''(x) + a(x)y''(x) + b(x)y(x) + c(x)y(x) = f(x),\\ &y(0) = p,\ y'(0) = q,\ y'(1) = r \end{aligned}$$

by using of an asymptotic numerical method.

In this paper, we discuss the existence, uniqueness and asymptotic estimates of solutions of the following third-order multi-point singularly perturbed boundary value problem (for short, BVP)

$$\varepsilon x'''(t) + f(t, x(t), x'(t), x''(t), \varepsilon) = 0, \quad 0 \leqslant t \leqslant 1,\ \ 0 < \varepsilon \ll 1, \tag{1.1}$$

$$\begin{aligned} &x(0, \varepsilon) = 0,\\ &ax'(0, \varepsilon) - bx''(0, \varepsilon) + \textstyle\sum_{i=1}^{n-2} \alpha_i x(\xi_i, \varepsilon) = A,\\ &cx'(1, \varepsilon) + dx''(1, \varepsilon) + \textstyle\sum_{i=1}^{n-2} \beta_i x(\eta_i, \varepsilon) = B, \end{aligned} \tag{1.2}$$

where $a, b, c, d \geqslant 0$, $A, B \in R$, $a + b > 0$, $c + d > 0$, $\alpha_i \leqslant 0$, $\beta_i \leqslant 0$, $i = 1, 2, \ldots, n-2$, $0 < \xi_1 < \xi_2 < \cdots < \xi_{n-2} < 1$, and $0 < \eta_1 < \eta_2 < \cdots < \eta_{n-2} < 1$.

In order to study BVP (1.1), (1.2), we first discuss the following nonlinear multi-point boundary value problem

$$x'''(t) + f(t, x(t), x'(t), x''(t)) = 0, \quad 0 \leqslant t \leqslant 1, \tag{1.3}$$