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J. Differential Equations 215 (2005) 268–319

Journal of Differential Equations

www.elsevier.com/locate/jde

Boundedness, global existence and continuous dependence for nonlinear dynamical systems describing physiologically structured populations

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Received 15 June 2004; revised 19 October 2004

Available online 15 December 2004

Abstract

The paper is aimed as a contribution to the general theory of nonlinear infinite dimensional dynamical systems describing interacting physiologically structured populations. We carry out continuation of local solutions to maximal solutions in a functional analytic setting. For maximal solutions we establish global existence via exponential boundedness and by a contraction argument, adapted to derive uniform existence time. Moreover, within the setting of dual Banach spaces, we derive results on continuous dependence with respect to time and initial state.

To achieve generality the paper is organized top down, in the way that we first treat abstract nonlinear dynamical systems under very few but rather strong hypotheses and thereafter work our way down towards verifiable assumptions in terms of more basic biological modelling ingredients that guarantee that the high level hypotheses hold.

Keywords: Structured population; Boundedness; Continuation; Continuous dependence; Global existence; Interaction variable; Nonlinear dynamical system; Weak*

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doi:10.1016/j.jde.2004.10.025

1. Introduction

1.1. Aims

The main aim of this paper is to present several new results concerning the general theory of nonlinear physiologically structured population models. Traditionally, see e.g. [30,24] and the references in both, the dynamics of such populations are described by partial differential equations (PDE), but more recently, see [10,12] by constructing next state operators, which define a nonlinear semigroup. Following the second approach, we here extend the local constructions of [10], give conditions for the existence of global solutions in terms of exponential bounds and establish continuity properties. For steady-state analysis for structured populations, see [11] (general structured populations), [15,16] (application to a cannibalism model) and for numerical approaches see [2–4,8,20].

An essential tool for constructing local next state operators is the so-called *method of interaction variables*, which consists basically in splitting a quasilinear problem into a linear problem and (coupled to that) a fixed point problem. After having outlined this method, we start by establishing a linear theory, which has a biological interpretation by itself: it describes the population dynamics, when conditions are such that interactions can be ignored. In particular, we illustrate how the mathematical theory of adjoint semigroups provides a natural framework for the investigation of continuity properties. More generally, the motivation for the use of duality is the combination of a general and natural population state space and a convenient space, the space of continuous functions vanishing at infinity, to work with. We present examples of semigroups representing a population evolution that are the "not strongly continuous adjoint" of a strongly continuous semigroup. In view of the coupling to a fixed point problem, the central object in our linear theory is a linear semigroup, which is not parametrized by time, but more generally by functions of time, which we call *inputs*. In this setting, we generalize the well-known fact, that the adjoint semigroup of a strongly continuous semigroup is continuous in the weak* topology, see [14] or [6], to semigroups with infinite dimensional parameters. In the classic [19, Section 10.10] treats aspects of *n*-parameter semigroups, but does not contain duality results.

Coming to the nonlinear problem, we find that continuous dependence can quickly be deduced from the corresponding properties of the linear problem. Moreover we demonstrate how, at an abstract level, under very few assumptions a fairly general local construction can be extended to maximal time intervals, such that one gets a *nonlinear semiflow*. Once established, the semiflow properties provide a framework for investigating global existence and qualitative behaviour.

From a mathematical point of view, the closest kin to the dynamical systems considered here are generated by differential equations with state-dependent delay. In fact we are dealing with "translation along orbits of ordinary differential equation" semigroups provided with nonlocal boundary conditions. The special feature is that both, the direction of the orbits and the speed of translation incorporate nonlinearities (it is this property that prevents us from applying the theory developed by Marcus and Mizel [23]). Our hope is that in the long run the kind of tools and results developed