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J. Differential Equations 215 (2005) 429–447
Journal of Differential Equations
www.elsevier.com/locate/jde

Local existence and Gevrey regularity of 3-D Navier–Stokes equations with ℓ_p initial data

Animikh Biswas[*, 1]

Department of Mathematics & Statistics, University of North Carolina at Charlotte, 9201 University City Blvd., Charlotte NC-28223, USA

Received 25 June 2004; revised 3 December 2004
Available online 26 February 2005

Abstract

We obtain local existence and Gevrey regularity of 3-D periodic Navier–Stokes equations in case the sequence of Fourier coefficients of the initial data is in ℓ_p $(p < 3/2)$. The ℓ_p norm of the sequence of Fourier coefficients of the solution and its analogous Gevrey norm remains bounded on a time interval whose length depends only on the size of the body force and the ℓ_p norm of the Fourier coefficient sequence of the initial data. The control on the Gevrey norm produces explicit estimates on the analyticity radius of the solution as in Foias and Temam (J. Funct. Anal. 87 (1989) 359–369). The results provide an alternate approach in estimating the space-analyticity radius of solutions to Navier–Stokes equations than the one presented by Grujić and Kukavica (J. Funct. Anal. 152 (1998) 447–466).

MSC: 35Q30; 35Q35; 76D05

Keywords: Navier–Stokes equations; Gevrey regularity

1. Introduction

In this article, we consider the Navier–Stokes equations (NSE) with space-periodic boundary condition. A method for estimating the space-analyticity radius of solutions of NSE in this setup was introduced by Foias and Temam in [FT]. The basic idea

* Fax: +1 704 687 6415.
E-mail address: abiswas@uncc.edu.

[1] Supported in part by NSF Grant number DMS–0074460 and Junior Summer Research Fellowship (2003) at UNC, Charlotte.

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doi:10.1016/j.jde.2004.12.012

in [FT] of interpolating between a suitably defined analyticity (Gevrey) norm and a Sobolev norm leads to a very simple energy method, eliminating the need of traditional estimates on higher order derivatives (for example, as in [M]). This also provides an explicit estimate of the radius of analyticity in terms of the Sobolev H^1-norm of the initial data and the size of the forcing term. Subsequently, in [GK], Grujić and Kukavica provided an estimate of the analyticity radius, where they assumed the initial data to be in L^q $(q > 3)$. Unlike in [FT], instead of estimating the Gevrey norm directly, Grujić and Kukavica achieve the relevant estimates on the analyticity radius by interpolating between the L^q norm of the initial data and the L^q norm of the complexified solution.

As mentioned before, we consider the 3-D NSE with space-periodic boundary condition. In this setup, the NSE can be reformulated in terms of its Fourier coefficients. The resulting system can be regarded as a nonlinear evolution equation in an appropriate sequence space. This is the so-called wavevectors formulation of the NSE (see [F] for a detailed exposition). We assume that the initial data is such that the ℓ_p $(p < \frac{3}{2})$ norm of its sequence of Fourier coefficients (which henceforth will be referred to as the ℓ_p norm of the periodic function) is finite. By employing only elementary Functional Analytic techniques which completely bypasses the Sobolev inequalities, we prove that there exists a local in time solution of the 3D-NSE with bounded ℓ_p and Gevrey norms. It should be noted here that the Hausdorff–Young inequality states that for $1 \leqslant p \leqslant 2$, the ℓ_p norm of a periodic function dominates its L^q norm, where q is the Hölder conjugate of p. Thus our assumption on the initial data is stronger than that in [GK] in case of estimate of analyticity radius, and that of [FK] and [GM] in case of local existence results. However, since we control the ℓ_p norm of the solution, our conclusion is also similarly strengthened. Consequently, the results in [FK,GK] or [GM] do not directly imply the results presented here. Moreover, our results provide generalization of those obtained using energy methods by Foias (see [F]), where it was assumed that the initial data is in ℓ_1. The proof given here employs fixed point methods and is motivated by [FK,K,GM].

The paper is organized as follows. In Section 2, we establish notation and discuss some preliminary material. In Section 3, we first obtain a local in time solution which is bounded in ℓ_p $(p < \frac{3}{2})$ norm, in case the initial data also belongs to ℓ_p $(p < \frac{3}{2})$. Subsequently, we show that this solution is regular. Finally, in Section 3, we obtain local in time solution, which is bounded in the Gevrey norm. This provides an alternative approach to that of [GK] and is more in the spirit of the results in [FT]. The treatment here is essentially self-contained and elementary.

2. Notation and preliminaries

We consider the Navier–Stokes equations of viscous incompressible fluids in $\Omega = [0, L]^3$ with space periodic boundary condition:

$$\frac{\partial \mathbf{u}}{\partial t}(x,t) - \nu\Delta\mathbf{u} + (\mathbf{u}\cdot\nabla)\mathbf{u} + \nabla p = \mathbf{f}, \tag{2.1}$$

$$\nabla\cdot\mathbf{u} = 0. \tag{2.2}$$