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Stability analysis of neutral type systems in Hilbert space

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Abstract

The asymtoptic stability properties of neutral type systems are studied mainly in the critical case when the exponential stability is not possible. We consider an operator model of the system in Hilbert space and use recent results on the existence of a Riesz basis of invariant finite-dimensional subspaces in order to verify its dissipativity. The main results concern the conditions of asymptotic non-exponential stability. We show that the property of asymptotic stability is not determinated only by the spectrum of the system but essentially depends on the geometric spectral characteristic of its main neutral term. Moreover, we present an example of two systems of neutral type which have both the same spectrum in the open left-half plane and the main neutral term but one of them is asymptotically stable while the other is unstable.

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Keywords: Neutral type systems; Exponential stability; Strong stability; Infinite dimensional systems

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1. Introduction

A number of applied problems from physics, mechanics, biology and other fields can be described by partial differential or delay differential equations. This leads to the construction and study of infinite-dimensional dynamical systems. In our work we are interested in stability theory of systems with a delayed argument. It is almost impossible to overview the huge literature on the subject, so we only cite some monographs [3,9,13,16,17] which are among the main references. The most used methods and approaches are developed for retarded differential equations while the case of neutral type systems remains more difficult and less studied so far. Our attention is attracted by the fact that in the case of neutral type systems one meets two essentially different type of stability: exponential and strong asymptotic non-exponential stability. The last type of stability is impossible for retarded systems, but may occur for neutral type systems. In particular as it is shown in [5] for a high-order differential equation of neutral type the smooth solutions decay essentially slower that exponential, namely as function $1/t^{\beta}, \beta > 0$. One of the explanations of this fact is that a neutral type equation may have an infinite sequence of roots of the characteristic equation with negative real parts approaching to zero. It is obvious that in such a case the equation is not exponentially stable and one needs more subtle methods in order to characterize this type of asymptotic stability.

Our approach is based on the general theory of C_0-semigroups of linear bounded operators (see e.g. [31]).

Let us give the precise description of the system and the operator model under consideration. We study the following neutral type system:

$$\dot{z}(t) = A_{-1}\dot{z}(t-1) + \int_{-1}^{0} A_2(\theta)\dot{z}(t+\theta)\,\mathrm{d}\theta + \int_{-1}^{0} A_3(\theta)z(t+\theta)\,\mathrm{d}\theta, \tag{1}$$

where A_{-1} is constant $n \times n$-matrix, $\det A_{-1} \neq 0$, A_2, A_3 are $n \times n$-matrices whose elements belong to $L_2(-1, 0)$. This equation occurs, for example, when a system of neutral type is stabilized. Even if the initial system contains pointwise delays only, then the set of natural feedback laws contains distributed delays (see e.g. [20,21]), so the corresponding closed-loop system takes the form (1).

We do not consider here the case of mixed retarded-neutral type systems, i.e. when $A_{-1} \neq 0, \det A_{-1} = 0$, and limit ourselves to one principal neutral term.

One of the main questions for the construction of an operator model and a corresponding dynamical system is the choice of the phase space. In [13], the framework is based on the description of the neutral type system in the space of continuous functions $C([-1, 0]; \mathbb{C}^n)$. The essential result in this framework is that the exponential stability is characterized by the condition that the spectrum is in the open left-half plane and bounded away from the imaginary axis (see also [14, Theorem 6.1]). The case when the spectrum is not bounded away from the imaginary axis is much more complicated. It has been shown in [12] that a linear neutral differential equation can have unbounded solutions even though the associated characteristic equation has only purely imaginary