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# Invariant manifolds of dynamical systems close to a rotation: Transverse to the rotation axis

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**Abstract**

We consider one parameter families of vector fields depending on a parameter $\varepsilon$ such that for $\varepsilon = 0$ the system becomes a rotation of $\mathbf{R}^2 \times \mathbf{R}^n$ around $\{0\} \times \mathbf{R}^n$ and such that for $\varepsilon > 0$ the origin is a hyperbolic singular point of saddle type with, say, attraction in the rotation plane and expansion in the complementary space. We look for a local subcenter invariant manifold extending the stable manifolds to $\varepsilon = 0$. Afterwards the analogous case for maps is considered. In contrast with the previous case the arithmetic properties of the angle of rotation play an important role.
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## 1. Introduction and statement of the main results

First we consider one-parameter families of vector fields on $\mathbf{R}^{2+n}$ such that, for a value of the parameter, the system becomes $-\beta x_2 \partial/\partial x_1 + \beta x_1 \partial/\partial x_2$, with $\beta \neq 0$, where $x = (x_1, x_2)$ denote coordinates on $\mathbf{R}^2$; let $z$ denote coordinates on $\mathbf{R}^n$.

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More concretely, we study families of vector fields $X_\varepsilon$ defined in some neighbourhood of the origin by differential equations of the following form:

$$X_\varepsilon : \begin{cases} x_1' = -\beta x_2 + \varepsilon(a_{11}x_1 + a_{12}x_2 + a_{13}\cdot z) + \varepsilon O(|(x_1, x_2, z)|^2) + O(\varepsilon^2), \\ x_2' = \beta x_1 + \varepsilon(a_{21}x_1 + a_{22}x_2 + a_{23}\cdot z) + \varepsilon O(|(x_1, x_2, z)|^2) + O(\varepsilon^2), \\ z' = \varepsilon(a_{31}\cdot x_1 + a_{32}\cdot x_2 + a_{33}\cdot z) + \varepsilon O(|(x_1, x_2, z)|^2) + O(\varepsilon^2), \end{cases} \tag{1}$$

where $\varepsilon \geqslant 0$ is a real parameter. We have used the following notation: if $a$ is a linear map and $v$ is a vector, $a \cdot v$ denotes the image of $v$. In the variables $x = x_1 + ix_2$, $\bar{x} = x_1 - ix_2$ Eq. (1) becomes

$$X_\varepsilon : \left.\begin{cases} x' = i\beta x + \varepsilon(b_{11}x + b_{12}\bar{x} + b_{13}\cdot z) \\ \bar{x}' = -i\beta \bar{x} + \varepsilon(\bar{b}_{12}x + \bar{b}_{11}\bar{x} + \bar{b}_{13}\cdot z) \\ z' = \varepsilon(b_{31}\cdot x + \bar{b}_{31}\cdot \bar{x} + a_{33}\cdot z) \end{cases}\right\rangle + \varepsilon O(|(x, \bar{x}, z)|^2) + O(\varepsilon^2), \tag{2}$$

where $b_{11} = (a_{11} + ia_{21} - ia_{12} + a_{22})/2$, $b_{12} = (a_{11} + ia_{21} + ia_{12} - a_{22})/2$, $b_{13} = a_{13} + ia_{23}$ and $b_{31} = (a_{31} - ia_{32})/2$. Since $\beta \neq 0$, we can simplify the linear terms by the change of variables $(x, \bar{x}, z) \mapsto (u, \bar{u}, w)$ defined by

$$\begin{cases} u = x + \varepsilon \frac{1}{i\beta}(b_{12}\bar{x} + b_{13}\cdot z), \\ \bar{u} = \text{complex conjugate of the first line}, \\ w = z + \varepsilon \frac{1}{i\beta}(-b_{31}\cdot x + \bar{b}_{31}\cdot \bar{x}). \end{cases} \tag{3}$$

It is then calculated that

$$\left.\begin{cases} u' = i\beta u + \varepsilon b_{11} u \\ \bar{u}' = -i\beta \bar{u} + \varepsilon \bar{b}_{11}\bar{u} \\ w' = \varepsilon a_{33}\cdot w \end{cases}\right\rangle + \varepsilon O(|(u, \bar{u}, w)|^2) + O(\varepsilon^2). \tag{4}$$

Note that the $O(\varepsilon^2)$ terms may contain constant and linear terms in $(u, \bar{u}, w)$. This kind of families shows up in the unfolding of the Hopf-zero singularity: see [5,7] for motivation, history and description of the problem. One is interested in the case when $\mathrm{Re}\, b_{11} < 0$ (that is: $a_{11} + a_{22} < 0$) and when all eigenvalues of $a_{33}$ have positive real part. In this case, for $\varepsilon > 0$, the system is, roughly speaking, contracting in the $(x_1, x_2)$-direction and expanding in the $z$-direction. The conditions above imply, using the implicit function theorem, that there is a hyperbolic singular point $p_\varepsilon = (x_\varepsilon, \bar{x}_\varepsilon, z_\varepsilon) = (O(\varepsilon^2), O(\varepsilon^2), O(\varepsilon))$ with an unstable manifold $W_\varepsilon^s$ of dimension two and a stable manifold $W_\varepsilon^u$ of dimension $n$. For the invariant manifold theory see, for instance, [10]. We make a translation in such a way that $p_\varepsilon \equiv 0$ becomes the origin. We can express these invariant manifolds locally as invariant graphs.

The analogous setting for maps consists of families of diffeomorphisms that behave as time one maps of equations of the form (2). More precisely, diffeomorphisms of

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