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Propagation and its failure in a lattice delayed differential equation with global interaction☆

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Abstract

We study the existence, uniqueness, global asymptotic stability and propagation failure of traveling wave fronts in a lattice delayed differential equation with global interaction for a single species population with two age classes and a fixed maturation period living in a spatially unbounded environment. In the bistable case, under realistic assumptions on the birth function, we prove that the equation admits a strictly monotone increasing traveling wave front. Moreover, if the wave speed does not vanish, then the wave front is unique (up to a translation) and globally asymptotic stable with phase shift. Of particular interest is the phenomenon of "propagation failure" or "pinning" (that is, wave speed $c = 0$), we also give some criteria for pinning in this paper.

MSC: 34K30; 35B40; 35R10; 58D25

Keywords: Lattice delayed differential equation; Global interaction; Traveling wave front; Existence; Uniqueness; Asymptotic stability; Propagation failure; Pinning

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1. Introduction

In recent years, spatially non-local differential equations have attracted the interest of more and more researchers. In the context of population biology, So et al. [22] recently derived the following delayed reaction–equation model:

$$w_t = Dw_{xx} - dw + \int_{-\infty}^{\infty} b(w(t-r, y)) f(x-y)\, dy, \tag{1.1}$$

which describes the evolution of the adult population of a single species population with two age classes and moves around in a unbounded one-dimensional spatial domain. Here $D > 0$ and $d > 0$ denote the diffusion rate and death rate of the adult population, respectively, $r \geqslant 0$ is the maturation time for the species, b is related to the birth function, and the kernel function f describes the diffusion pattern of the immature population during the maturation process, and hence, depends also on the maturation delay. We refer to So et al. [22] for more details and some specific forms of f, obtained from integrating along characteristic of a structured population model, an idea from the work of Smith and Thieme [20]. See also [23] for a similar model and [11] for a survey on the history and the current status of the study of reaction–diffusion equations with non-local delayed interactions. Also explored in [22] is the existence of traveling wave fronts of (1.1) when the reaction term is of monostable type. When the reaction term is of bistable type, Ma and Wu [16] investigated the existence, uniqueness and stability of a traveling wave front of (1.1).

More recently, Weng et al. [24] also derived a discrete analog of (1.1) for a single species in one-dimensional patchy environment with infinite number of patches connected locally by diffusion. This lattice equation has the form

$$u_n' = D[u_{n+1} + u_{n-1} - 2u_n] - du_n + \sum_{i=-\infty}^{\infty} J(i) b(u_{n-i}(t-r)). \tag{1.2}$$

In this paper, we always assume that $J(i) = J(-i) \geqslant 0$, $\sum_i J(i) = 1$ and $\sum_i |i| J(i) < +\infty$, here and in what follows, $\sum_i$ denotes the sum over $i \in \mathbb{Z}$. We also assume that the birth function $b \in C^1(\mathbb{R})$ and there exists a constant $K > 0$ such that

$$b(0) = dK - b(K) = 0.$$

Therefore, (1.2) has at least two spatially homogeneous equilibria 0 and K.

We point out that non-local discrete equations also arise from other fields. For example, in studying the phase transition phenomena, discrete convolutions equations are used in, e.g., Bates et al. [1] and Bates and Chmaj [2] and the references therein. We point out that the non-local terms in the models of [1,2] are linear, while the non-local term in (1.2) is nonlinear.