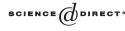


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J. Differential Equations 219 (2005) 40-77

Journal of Differential Equations

www.elsevier.com/locate/jde

Self-similar solutions of a semilinear parabolic equation with inverse-square potential $\stackrel{\text{\tiny\scale}}{\eqsim}$

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Received 2 August 2004; revised 23 June 2005

Available online 19 September 2005

Abstract

We investigate existence, nonexistence and asymptotical behaviour—both at the origin and at infinity—of radial self-similar solutions to a semilinear parabolic equation with inverse-square potential. These solutions are relevant to prove nonuniqueness of the Cauchy problem for the parabolic equation in certain Lebesgue spaces, generalizing the result proved by Haraux and Weissler [Non-uniqueness for a semilinear initial value problem, Indiana Univ. Math. J. 31 (1982) 167–189] for the case of vanishing potential.

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1. Introduction

In this paper we investigate existence, nonexistence and asymptotical behaviour of nonnegative solutions to the ordinary differential equation

$$(Pf')' + \left(\frac{c}{\xi^2} + \frac{1}{q-2}\right)Pf + P|f|^{q-2}f = 0$$
(1.1)

* Work partially supported through RTN Contract HPRN-CT-2002-00274.

0022-0396/ $\ensuremath{\$}$ - see front matter @ 2005 Elsevier Inc. All rights reserved. doi:10.1016/j.jde.2005.06.031

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in \mathbb{R}_+ , where

$$P(\xi) := \xi^{n-1} e^{\frac{\xi^2}{4}},$$
(1.2)

 $n \ge 3, q > 2$ and the coefficient c satisfies the inequality $0 < c < c_0, c_0 := \frac{(n-2)^2}{4}$ denoting the best constant in the Hardy inequality.

(a) Eq. (1.1) arises in the analysis of radial self-similar solutions to the semilinear parabolic equation with inverse-square potential

$$v_t = \Delta v + \frac{c}{r^2} v + |v|^{q-2} v$$
(1.3)

in $S := \mathbb{R}^n \times \mathbb{R}_+$ $(n \ge 3)$, where $r \equiv |x|$ and c, q are as above. Such solutions are of the form

$$v(x,t) = t^{-\frac{1}{q-2}} f(r/\sqrt{t}).$$

Upon substitution into (1.3), it is easily seen that the profile f satisfies Eq. (1.1) in \mathbb{R}_+ , where $\xi := r/\sqrt{t}$ and $' \equiv d/d\xi$.

In the following an important role is played by the roots:

$$\lambda = \lambda_{\pm} := 2 - n \pm 2\sqrt{c_0 - c} \tag{1.4}$$

of the equation

$$\lambda^{2} + 2(n-2)\lambda + 4c = 0 \tag{1.5}$$

(e.g., see Theorem 1.6 below; observe that $\lambda_{-} < 2 - n < \lambda_{+} < 0$). These roots naturally appear when we perform in Eq. (1.1) the change of unknown $f(\xi) = \xi^{\lambda/2}g(\xi)$; in fact, the choice $\lambda = \lambda_{\pm}$ gives the following equation for g:

$$(Hg')' - \sigma Hg + K|g|^{q-2}g = 0.$$
(1.6)

Here

$$H(\xi) := \xi^{\lambda + n - 1} e^{\frac{\xi^2}{4}} = \xi^{\lambda} P(\xi),$$
(1.7)

$$K(\xi) := \xi^{\frac{\lambda q}{2} + n - 1} e^{\frac{\xi^2}{4}} = \xi^{\frac{\lambda}{2}(q-2)} H(\xi),$$
(1.8)

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