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A discontinuous solution for an evolution compressible stokes system in a bounded domain

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Abstract

An evolution compressible Stokes system is studied in a bounded cylindrical region $Q = \Omega \times (0, T)$. The initial datum of pressure is assumed to have a jump at a specified curve $\mathcal{C}_0$ in Ω. As predicted by the Rankine–Hugoniot conditions, the pressure and velocity derivatives have jump discontinuities along the characteristic plane of the curve $\mathcal{C}_0$ directed by an ambient velocity vector. An explicit formula for the jump discontinuity is presented. The jump decays exponentially in time, more rapidly for smaller viscosities. Under suitable conditions of the data, a regularity of the solution is established in a compact subregion of Q away from the jump plane.

Keywords: Compressible Stokes equations; Discontinuous solutions

1. Introduction and main result

We are interested in constructing discontinuous solutions to the initial and boundary value problems of compressible Navier–Stokes equations in bounded regions provided the initial datum is discontinuous at a curve in the regions. The issues have been studied by several authors; for instance, among the references given in this paper, we refer to

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[3,4,7–11]. In [8,9] Hoff showed global existences to the Navier–Stokes equations for one and multi-dimensional compressible flows with discontinuous initial data and in [10] proved its global existence of the solutions having convecting singularity curves. The results obtained are thought of valuable contributions but we recall that the considered regions for the problems were the whole spaces R^n ($n = 2$ or 3). In [3,11] linearized equations or simple versions to the stationary compressible Stokes or Navier–Stokes equations were studied in an infinite strip in R^2 with discontinuous initial data.

On the other hand, many physical and engineering problems lead to the Navier–Stokes equations in domains with singular (or smooth) boundaries [2,6,15,16]. Hence it is wonder whether or not the results are true in bounded domains with boundaries having corners or edges, etc. In [4] discontinuous solutions to steady state viscous compressible Navier–Stokes equations were discussed on a rectangle of the plane.

In this paper, we start with a linearized evolution compressible Stokes system in a bounded region and will study the whole compressible Navier–Stokes system in a forthcoming paper. We show that if the pressure initial data has a jump at a curve $\mathcal{C}(0)$, then the pressure and velocity derivatives have jump discontinuities across a plane curve $\mathcal{C}(t)$ which is a transport of the curve $\mathcal{C}(0)$ by the ambient velocity $\mathbf{U}$. The governing system considered in this paper is

$$\begin{aligned}
\mathbf{u}_t - \mu\Delta\mathbf{u} + \nabla p &= \mathbf{f} && \text{in } Q,\\
\kappa\,(p_t + \mathbf{U}\cdot\nabla p) + \operatorname{div}\mathbf{u} &= g && \text{in } Q,\\
\mathbf{u} &= 0 && \text{on } \Sigma,\\
p &= h && \text{on } \Sigma_{\text{in}},\\
\mathbf{u}(\cdot,0) = \mathbf{u}_0,\ \ p(\cdot,0) &= p_0 && \text{on } \Omega.
\end{aligned} \tag{1.1}$$

Here $Q = \Omega \times (0, \mathrm{T})$ is a cylindrical domain, with a bounded plane domain Ω and a number $\mathrm{T} > 0$, $\Sigma = \Gamma \times [0, \mathrm{T}]$ is the lateral boundary of Q with the boundary Γ of Ω; $[\mathbf{u}, p]$ is the velocity and pressure variable; μ is the viscous number with $\mu > 0$, $\kappa := 1$ is the compressibility number and $\mathbf{U}$ is the ambient velocity vector; $\Sigma_{\text{in}} = \Gamma_{\text{in}} \times [0, \mathrm{T}]$ with $\Gamma_{\text{in}} = \{(x, y) \in \Gamma\ \ : \mathbf{U}\cdot\mathbf{n} < 0\}$ where $\mathbf{n}$ is the unit normal vector to Γ; $\mathbf{f}, g, h$ are given functions and $[\mathbf{u}_0, p_0]$ is the initial data.

For a simplicity of our setting, we let $\Omega = (-1, 1) \times (-1, 1)$ and $\mathbf{U} = [1, 0]$ in problem (1.1).

For derivation of the equations in (1.1), we refer to [2,12,13].

The continuity equation of (1.1) is a transport equation in pressure, so a discontinuity on either the inflow boundary of domain or an interior portion of the domain of the initial time can be developed into an interior of the flow region Q along the characteristic lines directed by the vector $\tilde{\mathbf{U}} = [\mathbf{U}, 1]$. Regarding this, we suspect that many mathematical and physical issues [2,15,16] are involved and has been resolved yet, so it will be worthwhile to investigate how the discontinuities develop into the region and what qualitative information we can have. In [3,11] it was shown that a linearized stationary compressible Stokes system admits an interior discontinuity solution by imposing a jump on that part of the boundary for which the ambient flow enters the region, and Chen and Xie [4] investigated a discontinuity for the solution to the