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Global structure instability of Riemann solutions of quasilinear hyperbolic systems of conservation laws: Rarefaction waves

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Abstract

This work is a continuation of our previous work (Kong, J. Differential Equations 188 (2003) 242–271) "Global structure stability of Riemann solutions of quasilinear hyperbolic systems of conservation laws: shocks and contact discontinuities". In the present paper we prove the global structure instability of the Lax's Riemann solution $u = U(\frac{x}{t})$, containing rarefaction waves, of general $n \times n$ quasilinear hyperbolic system of conservation laws. Combining the results in (Kong, 2003), we prove that the Lax's Riemann solution of general $n \times n$ quasilinear hyperbolic system of conservation laws is globally structurally stable if and only if it contains only non-degenerate shocks and contact discontinuities, but no rarefaction waves and other weak discontinuities.

MSC: 35L65; 35L45; 35L67

Keywords: Quasilinear hyperbolic system of conservation laws; Riemann solution; Rarefaction wave; Global structure instability

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1. Introduction

This work is a continuation of our previous work [8] “Global structure stability of Riemann solutions of quasilinear hyperbolic systems of conservation laws: shocks and contact discontinuities”. As in [8], we still consider the following quasilinear system of conservation laws:

$$\frac{\partial u}{\partial t}+\frac{\partial f(u)}{\partial x}=0, \tag{1.1}$$

where $u=(u_1,\dots,u_n)^T$ is the unknown vector function of (t,x), f: $\mathbb{R}^n\longrightarrow\mathbb{R}^n$ is a given C^3 vector function of u.

Suppose that on the domain under consideration, the system (1.1) is *strictly hyperbolic*, i.e., the Jacobi matrix $A(u)=\nabla f(u)$ possesses n distinct real eigenvalues:

$$\lambda_1(u)<\lambda_2(u)\cdots<\lambda_n(u). \tag{1.2}$$

For $i=1,\dots,n$, let $l_i(u)=(l_{i1}(u),\dots,l_{in}(u))$ (resp. $r_i(u)=(r_{i1}(u),\dots,r_{in}(u))^T$) be a left (resp. right) eigenvector corresponding to $\lambda_i(u)$. Without loss of generality, we may suppose that on the domain under consideration

$$l_i(u)\,r_j(u)\equiv\delta_{ij}\quad(i,j=1,\dots,n), \tag{1.3}$$

$$r_i^T(u)\,r_i(u)\equiv 1\quad(i=1,\dots,n), \tag{1.4}$$

where δ_{ij} stands for the Kronecker’s symbol.

Clearly, all $\lambda_i(u)$, $l_{ij}(u)$ and $r_{ij}(u)$ $(i,j=1,\dots,n)$ have the same regularity as $A(u)$, i.e., C^2 regularity.

Suppose furthermore that on the domain under consideration, each characteristic is either *genuinely nonlinear* in the sense of Lax (cf. [9]):

$$\nabla\lambda_i(u)r_i(u)\neq 0 \tag{1.5}$$

or *linearly degenerate* in the sense of Lax:

$$\nabla\lambda_i(u)r_i(u)\equiv 0. \tag{1.6}$$

Contrary to [8], in this paper we are interested in the global structure instability (in the sense of Definitions 1.1–1.2 in [8]) of the similarity solution of the Riemann