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Chaos in the beam equation

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Abstract

We show the existence of chaotic solutions for certain weakly damped linear beam equations with slowly periodic perturbations resting on weakly non-linear elastic bearings.

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Keywords: Beam equation; Weak non-linearities; Chaos

1. Introduction

This paper is a continuation of [3] where we proved the existence of a solution homoclinic to a small periodic solution of the equation

$$\begin{aligned} &u_{tt} + u_{xxxx} + \varepsilon\delta u_t + \varepsilon\mu h(x, \sqrt{\varepsilon}t) = 0\,,\\ &u_{xx}(0,\cdot) = u_{xx}(\pi/4,\cdot) = 0\,,\\ &u_{xxx}(0,\cdot) = -\varepsilon f(u(0,\cdot)), \quad u_{xxx}(\pi/4,\cdot) = \varepsilon f(u(\pi/4,\cdot))\,, \end{aligned} \tag{1}$$

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where $\varepsilon > 0$ and μ are sufficiently small parameters, $\delta > 0$ is a constant, $f \in C^2(\mathbb{R})$, $h \in C^2([0, \pi/4] \times \mathbb{R})$ and $h(x,t)$ is 1-periodic in t, provided an associated reduced equation has a homoclinic orbit. Eq. (1) describes vibrations of a beam resting on two identical bearings with purely elastic responses which are determined by f. The length of the beam is $\pi/4$. Since $\varepsilon > 0$, (1) is a semilinear problem.

In this paper we will prove the existence of infinitely many periodic as well as aperiodic, bounded solutions, under the same assumptions as in [3]. These solutions correspond to different modes of vibrations of (1). As a matter of fact, as in [3] we derive for $\varepsilon = 0$, $\mu = 0$ a reduced four-dimensional ordinary differential equation of (1) and prove that if the reduced equation has a homoclinic orbit and $\delta > 0$ is sufficiently large, the Smale horseshoe [16] can be embedded into the dynamics of (1).

Let us briefly recall some results related to Eq. (1). The undamped case ($\delta = 0$, $\mu = 0$ and $\varepsilon = 1$) was studied in [6,9] by using a variational method. In both papers, the problems studied are non-parametric.

The perturbation approach to the beam equation was earlier used by Holmes and Marsden [10]. Recent results in this direction are given in [4,7]. We note that problem (1) is more complicated than the one studied in [4,7,10], since in those papers the elastic response is distributed continuously along the beam, while in our case it is concentrated just at two end points of the beam. Moreover, the ε-smallness of the restoring force εf at the end points leads to a singularly perturbed problem in studying chaotic orbits of (1). The existence of homoclinic and chaotic solutions has also been proved in [1,11,12,15] for different partial differential equations and with different methods than ours.

2. Setting of the problem

First of all, we make the linear scale $t \leftrightarrow \sqrt{\varepsilon}t$ in (1), that is we take $u(x,t) \leftrightarrow u(x, \sqrt{\varepsilon}t)$ to get the equivalent problem

$$\begin{aligned} &u_{tt} + \tfrac{1}{\varepsilon}u_{xxxx} + \sqrt{\varepsilon}\delta u_t + \mu h(x,t) = 0\,, \\ &u_{xx}(0,\cdot) = u_{xx}(\pi/4,\cdot) = 0\,, \\ &u_{xxx}(0,\cdot) = -\varepsilon f(u(0,\cdot)), \quad u_{xxx}(\pi/4,\cdot) = \varepsilon f(u(\pi/4,\cdot))\,. \end{aligned} \tag{2}$$

By a (weak) solution of (2), we mean any $u(x,t) \in C([0,\pi/4] \times \mathbb{R})$ satisfying the identity

$$\begin{aligned} &\int_{-\infty}^{\infty}\int_0^{\pi/4} \left\{u(x,t)\left[v_{tt}(x,t) + \tfrac{1}{\varepsilon}v_{xxxx}(x,t) - \sqrt{\varepsilon}\delta v_t(x,t)\right]\right. \\ &\quad \left. + \mu h(x,t)v(x,t)\right\} dx\,dt \\ &\quad + \int_{-\infty}^{\infty} \{f(u(0,t))v(0,t) + f(u(\pi/4,t))v(\pi/4,t)\}\,dt = 0 \end{aligned} \tag{3}$$