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Regularity of ground state solutions of dispersion managed nonlinear schrödinger equations☆

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Abstract

We consider the dispersion managed nonlinear Schrödinger equation (DMNLS) in the case of zero residual dispersion. Using dispersive properties of the equation and estimates in Bourgain spaces we show that the ground state solutions of DMNLS are smooth. The existence of smooth solutions in this case matches the well-known smoothness of the solutions in the case of nonzero residual dispersion. In the case $x \in \mathbb{R}^2$ we prove that the corresponding minimization problem with zero residual dispersion has no solution.

Keywords: Dispersion managed nonlinear Schrödinger equation; Regularity; Ground states

1. Introduction and main result

Our work is motivated by the study of parametrically excited NLS with periodically varying dispersion coefficient

$$iu_t + D(t)u_{xx} + C(t)|u|^2u = 0,$$

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which arises as an envelope equation in the problem of an electromagnetic wave propagating in an optical waveguide. The balance between the dispersion and the nonlinearity in this equation is the key factor that determines the existence of stable pulses. In the last decade, a technique that uses fibers with alternating sections having opposite dispersion was introduced. This technology, called dispersion management, proved to be incredibly successful in producing stable, soliton-like pulses. The idea is to use rapidly varying dispersion with approximately zero mean and small nonlinearity in hope that the balance between the small residual dispersion and the small nonlinearity will produce a soliton-like solutions. There have been an enormous amount of technological advances in this direction with an array of numerical and phenomenological explanations and a recent theoretical understanding of the strong stability properties of the dispersion managed (DM) systems. The envelope equation that describes the propagation of electromagnetic pulses in optical fibers in the regime of strong dispersion management, derived by Gabitov and Turitsyn in 1996 [6,7] is a nonlinear Schrödinger equation with periodically varying coefficients. After rescaling the equation takes the form

$$iu_t + d(t)u_{xx} + \varepsilon(|u|^2 u + \alpha u_{xx}) = 0, \tag{1}$$

where t is the propagation distance, x is the retarded time and $d(t)$ is the mean-zero component of the dispersion, see [17]. Note that the average dispersion and nonlinearity are small compared to the local dispersion, which is a characteristic feature of the strong dispersion management. Performing Van der Pol transformation in (1) and averaging in the Hamiltonian we obtain the averaged variational principle

$$\langle H\rangle = \varepsilon \int_{-\infty}^{+\infty} \int_0^1 \left(\alpha|v_x|^2 - \frac{1}{2}|T(t)v|^4\right) dx\, dt \tag{2}$$

with the corresponding Euler–Lagrange equation (averaged), see [1,7]

$$iv_t + \varepsilon\alpha v_{xx} + \varepsilon\langle Q\rangle(v, v, v) = 0, \tag{3}$$

where

$$\langle Q\rangle(v_1, v_2, v_3) = \int_0^1 Q(v_1, v_2, v_3, t)\, dt.$$

Here $T(t)$ is the fundamental solution of $iu_t + d(t)u_{xx} = 0$ and

$$Q(v_1, v_2, v_3, t) = T^{-1}(t)(T(t)v_1 T(t)v_2 \overline{T(t)v_3}).$$

In [17] the existence of ground state solution for the averaged equations is proved, as well as an averaging result, which guarantees the existence of nearly periodic stable