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Nonlinear wave propagation through a stratified atmosphere

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Abstract

We examine the propagation of sound waves through a stratified atmosphere. The method of multiple scales is employed to obtain an asymptotic equation which describes the evolution of sound waves in an atmosphere with spatially dependant density and entropy fields. The evolution equation is an inviscid Burger-like equation which contains quadratic and cubic nonlinearities, and a curvature term all of which are functions of the space variables. A model equation is derived when the modulations of the signal in a direction transverse to the direction of propagation become significant.
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1. Introduction

The effectiveness of the geometrical optics method prompted many workers to extend the underlying ideas to wave motion (see, for example, Choquet-Bruhat [1], Hunter and Keller [2], Majda and Rosales [3], Cramer and Sen [4], Kluwick and Cox [5], and Srinivasan and Sharma [6]); the technique involves the introduction of slow and fast variables and the phase functions. These multiple scale expansions have been successfully employed

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in the context of ODEs [7], where these were introduced by Krylov–Bogoliubov as a variant of the methods employed earlier by Poincaré and Lindstedt to eliminate secular terms in the perturbation expansions of Celestial Mechanics. The precise scaling of the fast variables in comparison to the slow ones may vary depending on the problem under study. Recently, Kluwick and Cox [5] have considered fast variables with a different scaling than were hitherto considered; the need for this arose from the study of systems with mixed nonlinearities where the nonlinear distortions of the disturbance make their appearance noticeable over time scales of order $O(\varepsilon^{-2})$. Practically, any problem of acoustics takes place in the presence of a gravitational field, and as a consequence, the unperturbed state is not uniform. In problems of propagation over large distances in atmosphere or the ocean, these effects may be crucially important and produce amplification and refraction of the sound waves. Here, using the analytical apparatus developed by Kluwick and Cox [5], we study a quasilinear system with a source term that describes the propagation of sound waves in an atmosphere with spatially dependent density and entropy fields, and derive the transport equation for the high frequency wave amplitude in the leading order terms in the expansion. The quadratic and cubic nonlinear terms in the evolution equation strongly predict that smooth solutions develop shocks and result in the breakdown of the solution. Finally, a model equation is derived when the modulations of the signal in directions transverse to the direction of propagation become significant.

2. Basic equations

Equations describing the propagation of sound waves through a stratified fluid may be expressed in the form:

$$\begin{aligned}
&\mathbf{v}_{,t} + (\mathbf{v}.\boldsymbol{\nabla})\mathbf{v} + \rho^{-1}\nabla p = -\mathbf{g},\\
&\rho_{,t} + (\mathbf{v}.\boldsymbol{\nabla})\rho + \rho(\nabla.\mathbf{v}) = 0,\\
&s_{,t} + (\mathbf{v}.\boldsymbol{\nabla})s = 0,
\end{aligned}\tag{1}$$

where ρ is the density of the fluid, $p = p(\rho, s)$ the pressure, s the entropy, t the time, ∇ the gradient operator with respect to the space coordinates (x_1, x_2, x_3), and $\mathbf{g}$ the forcing function, which balances the initial conditions; a comma followed by the letter t denotes partial differentiation with respect to time, t. The reference state is characterized by a flow field at rest, $\mathbf{v} = \mathbf{0}$, with spatially varying density and entropy fields, namely $\rho_0 = \rho_0(\mathbf{x})$ and $s_0 = s_0(\mathbf{x})$ with $\nabla p_0 + \rho_0\mathbf{g} = 0$, where the subscript '0' is used to characterize the unperturbed fluid in equilibrium. The governing system (1) can be cast into the form

$$\mathbf{U}_{,t} + A^k(\mathbf{U})\mathbf{U}_{,k} + \mathbf{F} = 0, \quad k = 1, 2, 3, \tag{2}$$

representing a hyperbolic quasilinear system of equations with source terms, which may be attributed to the influences of gravity. Here $\mathbf{U}$ and $\mathbf{F}$ are column vectors defined as $\mathbf{U} = (v_1, v_2, v_3, \rho, s)'$ and $\mathbf{F} = (g_1, g_2, g_3, 0, 0)'$, respectively, with a prime denoting trans-

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