Available online at www.sciencedirect.com
SCIENCE DIRECT®
J. Math. Anal. Appl. 311 (2005) 69–75
Journal of MATHEMATICAL ANALYSIS AND APPLICATIONS
www.elsevier.com/locate/jmaa

On vector variational-like inequality problems ☆

S.K. Mishra [a,*], M.A. Noor [b]

[a] *Department of Mathematics, Statistics and Computer Science, College of Basic Sciences and Humanities, G.B. Pant University of Agriculture and Technology, Pantnagar-263 145, India*
[b] *Etisalat College of Engineering, PO Box 980, Sharjah, United Arab Emirates*

Received 10 January 2005
Available online 31 May 2005
Submitted by William F. Ames

Abstract

In this paper, we establish some relationships between vector variational-like inequality and vector optimization problems under the assumptions of α-invex functions. We identify the vector critical points, the weakly efficient points and the solutions of the weak vector variational-like inequality problems, under pseudo-α-invexity assumptions. These conditions are more general than those of existing ones in the literature. In particular, this work extends the earlier work of Ruiz-Garzon et al. [G. Ruiz-Garzon, R. Osuna-Gomez, A. Rufian-Lizan, Relationships between vector variational-like inequality and optimization problems, European J. Oper. Res. 157 (2004) 113–119] to a wider class of functions, namely the pseudo-α-invex functions studied in a recent work of Noor [M.A. Noor, On generalized preinvex functions and monotonicities, J. Inequal. Pure Appl. Math. 5 (2004) 1–9].

Keywords: Vector optimization; Pseudo-univex functions; Vector variational-like inequality problems

☆ This research is supported by the Department of Science and Technology, Ministry of Science and Technology, Government of India, under the SERC Fast Track Scheme for Young Scientists 2001–2002, through grant No. SR/FTP/MS-22/2001.
* Corresponding author.
E-mail addresses: shashikmishra@rediffmail.com (S.K. Mishra), noor@mscs.dal.ca, noormaslam@hotmail.com (M.A. Noor).

0022-247X/$ – see front matter
doi:10.1016/j.jmaa.2005.01.070

1. Introduction

The concept of vector variational inequality was introduced by Giannessi [1] in 1980. Since it has shown applications to a wide range of problems in various disciplines in the natural and social sciences, vector variational inequality problems have been generalized in various directions; in particular, vector variational-like inequality problems, see [2,12,14,16].

The role of generalized monotonicity of the operator in vector variational inequality problems corresponds to the role of generalized convexity of the objective function in the optimization problem. In recent years, several extensions and generalizations have been considered for classical convexity. A significant generalization of convex functions is that of invex functions introduced by Hanson [3]. For further developments in this direction see [4–6,11]. Weir and Mond [13] and Noor [7–9] have studied some basic properties of the preinvex functions and their role in optimization and variational-like inequality problems. Noor [8] has pointed out that the concept of invexity plays exactly the same role in variational-like inequality problems as the classical convexity plays in variational inequality problems, and has shown that the variational-like inequality problems are well defined in the setting of invexity.

Recently, Ruiz-Garzon et al. [12] established relationships between vector variational-like inequality and optimization problems, under the assumptions of pseudo-invexity. However, Ruiz-Garzon et al. [12] have obtained some results without invexity assumption on the underlying set while discussing variational-like inequality problems.

Recently Noor [9] has studied some properties of the α-*preinvex* functions and their differentials. Motivated by the work of Noor [9], we establish various relationships between generalized vector variational-like inequality problems and vector optimization problems under the assumption of pseudo-α-invex functions. In Section 2, we recall some definitions and preliminaries. In Section 3, we establish relationships between generalized vector variational-like inequality problems and vector optimization problems.

2. Preliminaries

The following convention for equalities and inequalities will be used throughout the paper. If $x = (x_1, x_2, \ldots, x_n)$ and $y = (y_1, y_2, \ldots, y_n) \in R^n$, we denote

$$x \leqq y \quad \text{iff} \quad x_i \leqslant y_i \quad \forall i = 1, 2, \ldots, n;$$

$$x \leq y \quad \text{iff} \quad x_i \leqslant y_i \quad \forall i = 1, 2, \ldots, n \text{ with } x \neq y;$$

$$x < y \quad \text{iff} \quad x_i < y_i \quad \forall i = 1, 2, \ldots, n; \quad \text{and}$$

$x \not< y$ is the negation of $x < y$.

Let X be a nonempty subset of R^n, $\eta : X \times X \to R^n$ be a continuous map and $\alpha : X \times X \to R_+ \setminus \{0\}$ be a bifunction. First of all, we recall some known results and concepts, see Noor [9].