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Solvability of initial-boundary value problems for bending of thermoelastic plates with mixed boundary conditions

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Abstract

Initial-boundary value problems for bending of a thermoelastic plate with transverse shear deformation are studied under the assumption that various parts of the boundary are subjected to different types of physical conditions. The unique solvability of these problems is established in spaces of distributions by means of a combination of the Laplace transform and variational methods.

Keywords: Thermoelastic plate; Initial-boundary value problem; Unique solvability; Variational method

1. Introduction

Theories of elastic plates are formulated and used in order to simplify the mathematical model by reducing it from a three-dimensional problem to a two-dimensional one. Another advantage of such theories is that they describe the essence of the mechanical process of bending by neglecting secondary, less important effects. What is referred to in

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the literature as the 'classical' model (Kirchhoff, 1850) is still being employed as a reasonable tool in various practical situations, but it does not produce the range of information that newer theories do. Given the increasing need for accuracy placed on research by today's sophisticated technology, such new models are becoming more and more popular with practitioners, engendering a demand for a thorough study of their mathematical nature. For example, the theory of bending of plates with transverse shear deformation [1] produces good approximations not only for the bending and twisting moments, but also for the shear force and the displacement field. The model discussed in [1] has subsequently been generalized further, to take additional account of thermal effects [2].

In this paper, we consider the time-dependent bending of a thin elastic plate subjected to external forces and moments and internal heat sources, together with homogeneous initial conditions and mixed boundary conditions. Analytic considerations lead to a variational formulation of the problems, which is rigorously investigated in spaces of distributions. The Laplace transformation is used to change the model into an elliptic boundary value problem that depends on a parameter. After the latter is solved by means of function-analytic techniques, conclusions are drawn about the well-posedness of the original, non-stationary problem; specifically, it is shown that the problem has a unique weak solution which depends continuously (in a suitable norm) on the data. The model is thus readied for numerical computation.

The corresponding results in the absence of thermal effects were obtained in [3–7].

2. Formulation of the problem

Consider a thin, homogeneous and isotropic elastic plate of thickness $h_0 = \mathrm{const} > 0$, which occupies a region $\bar{S} \times [-h_0/2, h_0/2]$ in $\mathbb{R}^3$, where S is a domain in $\mathbb{R}^2$. The displacement vector at a point x' in this region at $t \geqslant 0$ is denoted by $v(x',t) = (v_1(x',t), v_2(x',t), v_3(x',t))^{\mathrm{T}}$, where the superscript T means matrix transposition. The temperature in the plate is denoted by $\theta(x',t)$. Let $x' = (x, x_3)$, $x = (x_1, x_2) \in \bar{S}$. In plate models with transverse shear deformation it is assumed [1] that

$$v(x',t) = \big(x_3 u_1(x,t), x_3 u_2(x,t), u_3(x,t)\big)^{\mathrm{T}}.$$

When thermal effects are significant, we also take into account the "averaged" temperature across thickness, defined by [2]

$$u_4(x,t) = \frac{1}{h^2 h_0} \int_{-h_0/2}^{h_0/2} x_3 \theta(x, x_3, t)\, dx_3, \qquad h^2 = \frac{h_0^2}{12}.$$

Here the factor $1/h^2$ has been introduced purely for reasons of convenience. Then the vector-valued function $U(x,t) = (u(x,t)^{\mathrm{T}}, u_4(x,t))^{\mathrm{T}}$, where

$$u(x,t) = \big(u_1(x,t), u_2(x,t), u_3(x,t)\big)^{\mathrm{T}},$$

satisfies the equation

$$\mathcal{B}_0 \partial_t^2 U(x,t) + \mathcal{B}_1 \partial_t U(x,t) + \mathcal{A} U(x,t) = \mathcal{Q}(x,t), \qquad (x,t) \in G, \tag{1}$$