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Corrigendum

Corrigendum to "On norms of composition operators acting on Bergman spaces" [J. Math. Anal. Appl. 291 (2004) 189–202]

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Abstract

A theorem and a corollary in the paper cited in the title were stated incorrectly, as was pointed out by Christopher Hammond. We now state correctly and prove both of them. These results still generalize and explain the geometric meaning of the Cowen–Hurst norm formula. We also include additional references and provide an example relevant for further study.

Keywords: Composition operator; Bergman spaces; Operator norm; Essential norm; Angular derivative

In [8] we established several norm estimates and exact norm formulas for composition operators acting on a general Bergman space A^p of the unit disk, easy to adapt to the weighted Bergman or Hardy spaces. In particular, we were concerned with a generalization and geometric meaning of the well-known formula due to Cowen [2] and Hurst [7] for the norm of a composition operator induced by an affine map.

Christopher Hammond has observed that both Theorem 6 and Corollary 9 in [8] were incorrect as stated (for linear fractional maps). However, it is not difficult to verify that they remain true for linear symbols, as in [2] or [7]. Our geometric interpretation is thus still

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valid and the proofs are simple. Having the result for A^p instead of just A^2 also appears to be a true generalization. Namely, even though knowing the norm of C_φ as an H^2 operator readily yields an analogous formula for H^p because of the properties of Blaschke products (see, for example, Proposition 2.15 of [4]), this phenomenon does not carry over directly to the Bergman spaces in any obvious way.

We first state and prove the correct version of Theorem 6 from [8], using the same notation as in [2], in Chapter 9 of [3], or in [7]. It should be noted that

$$\left(1+|s|^2-|t|^2\right)^2-4|s|^2=\left(1-|s|^2+|t|^2\right)^2-4|t|^2,$$

so the quantity in our formula coincides with the expression that appears in the three sources cited.

Theorem. *Let $\varphi(z)=sz+t$, $|s|+|t|<1$; in this case, $\varphi(\mathbb{D})$ is the Euclidean disk $D(t,|s|)$ that coincides with some pseudo-hyperbolic disk $\Delta(a,r)$, $a\in\mathbb{D}$, $0<r<1$. Then the norm of C_φ as an operator on the Bergman space A^p, $1\leqslant p<\infty$, equals*

$$\|C_\varphi\|=\left(\frac{r}{|s|}\right)^{2/p}=\left(\frac{2}{1+|s|^2-|t|^2+\sqrt{(1+|s|^2-|t|^2)^2-4|s|^2}}\right)^{2/p}. \tag{1}$$

Proof. After the change of variable $w=sz+t$, we get

$$\int_{\mathbb{D}}\left|f\left(\varphi(z)\right)\right|^p dA(z)=\frac{1}{|s|^2}\int_{D(t,|s|)}\left|f(w)\right|^p dA(w)=\frac{1}{|s|^2}\int_{\Delta(a,r)}|f|^p\,dA$$
$$\leqslant\left(\frac{r}{|s|}\right)^2\int_{\mathbb{D}}|f|^p\,dA,$$

with equality when $f=I_a(\mathbf{1})=(-\varphi_a')^{2/p}$, in view of Lemma 5 of [8] and the basic property of the isometries I_a. □

The error in the earlier proof occurred in the change of variable. The problem lies in the fact that even though φ_a is its own inverse and $\varphi_a'=\varphi'$, rewriting the Jacobian as was done in [8] was not justified.

We should also point out that Theorem 6 as announced in [8] does not hold in full generality. Namely, the initial incorrect statement suggested that the norms of two operators C_φ and C_ψ induced by linear fractional maps should be equal as long as the image disks $\varphi(\mathbb{D})$ and $\psi(\mathbb{D})$ coincide. This is false as shown by the following example due to Hammond [6]. Consider the linear fractional self maps of the disk given by the formulas

$$\varphi(z)=\frac{z+1}{z+2},\qquad \psi(z)=\frac{z+1}{3}.$$

Then $\varphi(\mathbb{D})=\psi(\mathbb{D})=D(1/3,1/3)$. However, the norms of the two operators acting on A^2 are different! According to, e.g., Theorem 1 of [8], on A^2 we get

$$\|C_\varphi\|\geqslant\frac{1}{1-|\varphi(0)|^2}=\frac{4}{3},$$