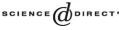


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J. Math. Anal. Appl. 312 (2005) 576-595

Journal of MATHEMATICAL ANALYSIS AND APPLICATIONS

www.elsevier.com/locate/jmaa

## Blow-up rate estimates for a doubly coupled reaction–diffusion system ☆

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Received 29 November 2004 Available online 12 April 2005 Submitted by H.A. Levine

## Abstract

This paper deals with a reaction-diffusion system with coupled nonlinear inner sources and a nonlinear boundary flux. Blow-up rates are determined for four different blow-up situations. The so-called characteristic algebraic system is introduced to get a very simple and clear description for the desired blow-up rate estimates. It is pointed out that one cannot directly use super and sub-solutions to establish blow-up rate estimates, since they do not share the same blow-up time in general. © 2005 Elsevier Inc. All rights reserved.

Keywords: Reaction-diffusion system; Characteristic algebraic system; Blow-up rate

## 1. Introduction

In this paper, we consider the following reaction–diffusion equations coupled via both nonlinear sources and a nonlinear boundary flux:

\* Supported by National Natural Science Foundation of China.

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<sup>0022-247</sup>X/\$ – see front matter @ 2005 Elsevier Inc. All rights reserved. doi:10.1016/j.jmaa.2005.03.046

$$\begin{cases} u_t = u_{xx} + u^{l_{11}} v^{l_{12}}, & v_t = v_{xx} + u^{l_{21}} v^{l_{22}}, \\ (x,t) \in (0,1) \times (0,T), \\ u_x(1,t) = (u^{p_{11}} v^{p_{12}})(1,t), & v_x(1,t) = (u^{p_{21}} v^{p_{22}})(1,t), & t \in (0,T), \\ u_x(0,t) = 0, & v_x(0,t) = 0, \\ u(x,0) = u_0(x), & v(x,0) = v_0(x), & x \in (0,1), \end{cases}$$
(1.1)

where  $l_{ij}$ ,  $p_{ij} \ge 0$  (i, j = 1, 2);  $u_0(x)$  and  $v_0(x)$  are smooth functions satisfying the compatible conditions. We deal with completely coupled cases only: it is required that at least one of  $l_{12}l_{21}$ ,  $l_{12}p_{21}$ ,  $p_{12}l_{21}$  and  $p_{12}p_{21}$  is positive.

Global existence and nonexistence of positive solutions of (1.1) can be found in [26,33]. It was proved that the solutions of (1.1) blow up in finite time if and only if at least one of the following conditions holds:  $l_{11} > 1$ ;  $l_{22} > 1$ ;  $p_{11} > 1$ ;  $p_{22} > 1$ ;  $p_{12}p_{21} > (1 - p_{11})(1 - p_{22})$ ;  $l_{12}l_{21} > (1 - l_{11})(1 - l_{22})$ ;  $l_{12}p_{21} > (1 - p_{11})(1 - l_{22})$ ;  $l_{12}p_{21} > (1 - l_{21})(1 - l_{22})(1 - l_{21})(1 - l_{22})$ ;  $l_{12}p_{21} > (1 - l_{21})(1 - l_{22})(1 - l_{21})(1 - l_{22}$ 

More special cases of (1.1) were studied by many authors. The heat equations coupled via a nonlinear boundary flux

$$\begin{cases} u_t = \Delta u, \quad v_t = \Delta v & \text{in } \Omega \times (0, T), \\ \frac{\partial u}{\partial \eta} = u^{p_{11}} v^{p_{12}}, \quad \frac{\partial v}{\partial \eta} = u^{p_{21}} v^{p_{22}} & \text{on } \partial \Omega \times (0, T), \end{cases}$$
(1.2)

were studied by Rossi [25], Pederson and Lin [21]. The blow-up rates of radial solutions of (1.2) with large initial data and  $\Omega = B_R$  were known as follows:

$$c \leq \sup_{B_R} u(\cdot, t)(T-t)^{\tilde{\alpha}_1} \leq C, \qquad c \leq \sup_{B_R} v(\cdot, t)(T-t)^{\beta_1} \leq C,$$
$$\tilde{\alpha}_1 = \frac{p_{12} + 1 - p_{22}}{2[p_{12}p_{21} - (1-p_{11})(1-p_{22})]}, \qquad \tilde{\beta}_1 = \frac{p_{21} + 1 - p_{11}}{2[p_{12}p_{21} - (1-p_{11})(1-p_{22})]}.$$

The case for more general domains was considered by Chen [1], and the case with  $p_{ii} = 0$  (i = 1, 2) of (1.2) was studied by Deng [3]. Scalar cases of (1.2) were well studied in [4,11–14].

The blow-up rates of radial solutions to the homogeneous Dirichlet problem of coupled reaction-diffusion equations

$$u_t = \Delta u + u^{l_{11}} v^{l_{12}}, \qquad v_t = \Delta v + u^{l_{21}} v^{l_{22}} \quad \text{in } B_R \times (0, T)$$
 (1.3)

were obtained by Zheng [35] and Wang [28] as

$$c \leq \sup_{B_R} u(\cdot, t)(T-t)^{\tilde{\alpha}_2} \leq C, \qquad c \leq \sup_{B_R} v(\cdot, t)(T-t)^{\beta_2} \leq C,$$
  
$$\tilde{\alpha}_2 = \frac{l_{12} + 1 - l_{22}}{l_{12}l_{21} - (1 - l_{11})(1 - l_{22})}, \qquad \tilde{\beta}_2 = \frac{l_{21} + 1 - l_{11}}{l_{12}l_{21} - (1 - l_{11})(1 - l_{22})}$$

The studies for scalar cases of (1.3) can be found in [6,8–10]. A special case of (1.3) with  $l_{ii} = 0$  (i = 1, 2) was discussed by Wang [30].

Fu and Guo [7], Wang [31] considered blow-up rates and sets for system (1.1) with  $l_{ii} = p_{ii} = 0$  (i = 1, 2). The other cases for (1.1) with  $l_{21} = p_{12} = l_{ii} = p_{ii} = 0$  (i = 1, 2) and  $l_{21} = l_{ii} = p_{ii} = 0$  (i = 1, 2) were studied by Wang also [29,32].

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