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# Large time behavior and energy relaxation time limit of the solutions to an energy transport model in semiconductors

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**Abstract**

In this paper, the global existence and the large time behavior of smooth solutions to the initial boundary value problem for the multi-dimensional energy transport model are studied. It is also proved that the solutions of the problem converge to an *isothermal* drift–diffusion model as energy relaxation time $\tau$ goes to 0 by compactness argument with the help of energy estimates and entropy inequality.


*Keywords:* Energy transport; Drift diffusion; Large time behavior; Energy relaxation time limit; Entropy inequality

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## 1. Introduction

In recent years, a class of strongly coupled parabolic systems with cross diffusion terms were derived from applied science. In real applications, due to more information included, such class of cross diffusion models describe the phenomena more clearly than the classical weakly coupled diffusion systems. But very few theoretical results have been obtained up to now. It is well known that if the system is not weakly coupled, no general theory like the results in [15] can be used directly. In fact, the structure is completely different from the weakly coupled case so that the usual method including the maximum principle and the regularity theory for parabolic equations cannot be used.

In the present paper, we will study a system, i.e. energy transport model, derived from semiconductor simulations. For more details of the energy transport model, we refer to [1,2,8,13,14,17]. The energy transport model is a degenerate quasi-linear cross diffusion parabolic system with principal part in divergence form. The common form of the energy transport model is governed by the system

$$\begin{aligned}
&\frac{\partial}{\partial t}\rho(\mu,T)+\operatorname{div}J_1=0,\\
&\frac{\partial}{\partial t}U(\mu,T)+\operatorname{div}J_2=\nabla V\cdot J_1+W(\mu,T)\quad \text{in } \Omega,\\
&\lambda^2\Delta V=\rho-C(x),
\end{aligned}\tag{1.1}$$

with

$$\begin{aligned}
J_1&=-L_{11}\left(\nabla\left(\frac{\mu}{T}\right)-\frac{\nabla V}{T}\right)-L_{12}\nabla\left(-\frac{1}{T}\right),\\
J_2&=-L_{21}\left(\nabla\left(\frac{\mu}{T}\right)-\frac{\nabla V}{T}\right)-L_{22}\nabla\left(-\frac{1}{T}\right),
\end{aligned}\tag{1.2}$$

where the parameters $\mu$ and $T$ are chemical potential of the electrons and the electron temperature respectively, $V$ is the electrostatic potential, $\rho(\mu,T)$ is the electron density, $U(\mu,T)$ is the density of the internal energy, $W(\mu,T)$ is the energy relaxation term satisfying $W(\mu,T)(T-T_0)\leqslant 0$, where the positive constant $T_0$ is the lattice temperature, $J_1$ is the carrier flux density, $J_2$ is the energy flux density, or heat flux, $L$ is the diffusion matrix, $\lambda$ is the scaled Debye length, and $C(x)$ is the doping profile which represents the background of the device. The expressions for $\rho$, $U$, $L$ and $W$ are constitutive relations. Various forms, corresponding to different models, are found in the literature.

In a parabolic band structure, the relations for $\rho(\mu,T)$ and $U(\mu,T)$ derived from the Boltzmann statistics are

$$\rho(\mu,T)=T^{\frac{3}{2}}\exp\left\{\frac{\mu}{T}\right\},\qquad U(\mu,T)=\frac{3}{2}\rho T.\tag{1.3}$$

Several authors have recently studied stationary energy transport models [4,6,10] and have obtained useful results. For the transient case, the first results on the existence of a weak solution and its large time behavior for a more general parabolic system were obtained by P. Degond et al. [7]. They employed semidiscretization in time and entropy function under the physically motivated Dirichlet–Neumann boundary conditions and initial