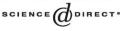


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Value sharing and differential equations

Ping Li

Department of Mathematics, University of Science & Technology of China, Hefei, Anhui 230026, PR China

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Abstract

Suppose that f is a nonconstant entire function and L[f] a linear differential polynomial in f with constant coefficients. In this paper, by considering the existence of the solutions of some differential equations, we find all the forms of entire functions f in most cases when f and L[f] share two values counting multiplicities jointly. This result generalize some known results due to Rubel–Yang and Li–Yang.

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1. Introduction and results

Let f(z) and g(z) be two nonconstant meromorphic functions, and c a finite complex number. If f(z) - c and g(z) - c have the same zeros counting multiplicities (ignoring multiplicities), then we say that f(z) and g(z) share the value c CM (IM). We say f(z) and g(z) share ∞ CM (IM) if 1/f and 1/g share 0 CM (IM). Let S be a subset of distinct elements in \mathbb{C} . Define

$$E_f(S) = \bigcup_{a \in S} \{ z \mid f(z) - a = 0, \text{ counting multiplicities} \},\$$

E-mail address: pli@ustc.edu.cn.

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$$\tilde{E}_f(S) = \bigcup_{a \in S} \{ z \mid f(z) - a = 0, \text{ ignoring multiplicities} \}.$$

We say that f(z) and g(z) share the set *S* CM (IM) provided that $E_f(S) = E_g(S)$ $(\tilde{E}_f(S) = \tilde{E}_g(S))$. It was shown [11] that if an entire function *f* and its derivative *f'* share two finite values *a*, *b* CM, then $f \equiv f'$. Mues–Steinmetz [8], and G. Gundersen [4] independently, improved this result. They proved that the conclusion remains to be valid if *f* share two values IM with *f'* only. This result has been generalized to the case that *f* share two values with a linear differential polynomial in *f* by several mathematicians, see, e.g., [1] and [9,10]. Li–Yang [7] considered the case that *f* share two values CM jointly with its derivative, and proved that if *f* and *f'* share the set $\{a_1, a_2\}$ CM, then *f* assume one of the following cases: (i) $f \equiv f'$; (ii) $f + f' \equiv a_1 + a_2$; (iii) $f \equiv c_1e^{cz} + c_2e^{-cz}$, with $a_1 + a_2 = 0$, where *c*, c_1 and c_2 are nonzero constants which satisfy $c^2 \neq 1$ and $c_1c_2 = \frac{1}{4}a_1^2(1 - c^{-2})$.

It is natural to ask what will be happen when f share two values jointly with its linear differential polynomial. In the present paper, we shall prove the following result.

Theorem 1. Suppose that f is a nonconstant entire function and

$$g = L_n[f] = b_{-1} + \sum_{i=0}^n b_i f^{(i)},$$
(1)

where b_i (i = -1, 0, 1, ..., n) are constants and $b_n \neq 0$. Let a_1 and a_2 be two distinct numbers in \mathbb{C} . If f and g share the set $\{a_1, a_2\}$ CM, then one of the following cases holds:

- (i) f = g;
- (ii) $f + g = a_1 + a_2;$
- (iii) $f = \frac{a_1+a_2}{2} + \frac{h_1}{2} + \frac{h_2}{2}$, $g = \frac{a_1+a_2}{2} \frac{1}{2}e^{\gamma}h_1 + \frac{1}{2}e^{\gamma}h_2$, where γ is an entire function such that $T(r, e^{\gamma}) = S(r, f)$ and h_1, h_2 are two entire functions satisfying $h_1h_2 = (\frac{a_1-a_2}{2})^2(1-e^{-2\gamma})$, and the following two differential equations:

$$\sum_{k=0}^{n} b_k h_1^{(k)} = -e^{\gamma} h_1, \qquad \sum_{k=0}^{n} b_k h_2^{(k)} = e^{\gamma} h_2.$$
(2)

Moreover, $2b_{-1} + (a_1 + a_2)(b_0 - 1) = 0$.

By considering the existence of the solutions of the two equations in (2), we also give the explicit forms of h_1 and h_2 for some special cases.

Theorem 2. Suppose that f is a nonconstant entire function and $g = L_n[f]$ is the differential polynomial defined in (1), where n = 1 or n is an even number. Let a_1 and a_2 be two distinct numbers in \mathbb{C} . If f and g share the set $\{a_1, a_2\}$ CM, then one of the following cases holds:

(i)
$$f = g;$$

(ii) $f + g = a_1 + a_2;$

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