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Exact controllability of the suspension bridge model proposed by Lazer and McKenna

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Abstract

In this paper we give a sufficient condition for the exact controllability of the following model of the suspension bridge equation proposed by Lazer and McKenna in [A.C. Lazer, P.J. McKenna, Large-amplitude periodic oscillations in suspension bridges: Some new connections with nonlinear analysis, SIAM Rev. 32 (1990) 537–578]:

$$\begin{cases} w_{tt} + cw_t + dw_{xxxx} + kw^+ = p(t,x) + u(t,x) + f(t,w,u(t,x)), & 0 < x < 1, \\ w(t,0) = w(t,1) = w_{xx}(t,0) = w_{xx}(t,1) = 0, & t \in \mathbb{R}, \end{cases}$$

where $t \geqslant 0$, $d > 0$, $c > 0$, $k > 0$, the distributed control $u \in L^2(0,t_1;L^2(0,1))$, $p:\mathbb{R}\times[0,1]\to\mathbb{R}$ is continuous and bounded, and the non-linear term $f:[0,t_1]\times\mathbb{R}\times\mathbb{R}\to\mathbb{R}$ is a continuous function on t and globally Lipschitz in the other variables, i.e., there exists a constant $l > 0$ such that for all $x_1, x_2, u_1, u_2 \in \mathbb{R}$ we have

$$\left\| f(t,x_2,u_2) - f(t,x_1,u_1) \right\| \leqslant l\left\{ \|x_2 - x_1\| + \|u_2 - u_1\| \right\}, \quad t \in [0,t_1].$$

To this end, we prove that the linear part of the system is exactly controllable on $[0,t_1]$. Then, we prove that the non-linear system is exactly controllable on $[0,t_1]$ for t_1 small enough. That is to say, the controllability of the linear system is preserved under the non-linear perturbation $-kw^+ + p(t,x) + f(t,w,u(t,x))$.

Keywords: Suspension bridge equation; Strongly continuous groups; Exact controllability

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1. Introduction

After the Tacoma Narrows Bridge collapsed on November 7, 1940, a lot of work have been done in the study of suspension bridge models. An important contribution is the work done by A.C. Lazer and P.J. McKenna in [6] and J. Glover et al. in [5] who proposed the following mathematical model for suspension bridges:

$$\begin{cases} w_{tt} + cw_t + dw_{xxxx} + kw^+ = p(t,x), \quad 0 < x < 1,\ t \in \mathbb{R}, \\ w(t,0) = w(t,1) = w_{xx}(t,0) = w_{xx}(t,1) = 0, \quad t \in \mathbb{R}, \end{cases} \tag{1.1}$$

where $d > 0$, $c > 0$, $k > 0$ and $p : \mathbb{R} \times [0,1] \to \mathbb{R}$ is continuous and bounded function acting as an external force.

The existence of bounded solutions of this model (1.1) and other similar equations has been carried out recently in [1–4,7,8]. To our knowledge, the exact controllability of this model under non-linear action of the control has not been studied before. So, in this paper we give a sufficient condition for the exact controllability of the following controlled suspension bridge equation:

$$\begin{cases} w_{tt} + cw_t + dw_{xxxx} + kw^+ = p(t,x) + u(t,x) + f(t,w,u(t,x)), \\ \quad 0 < x < 1, \\ w(t,0) = w(t,1) = w_{xx}(t,0) = w_{xx}(t,1) = 0, \quad t \in \mathbb{R}, \end{cases} \tag{1.2}$$

where the distributed control u belong to $L^2(0,t_1; L^2(0,1))$ and $f : [0,t_1] \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is a continuous function on t and globally Lipschitz in the other variables, i.e., there exists a constant $l > 0$ such that for all $x_1, x_2, u_1, u_2 \in \mathbb{R}$ we have

$$\left\| f(t,x_2,u_2) - f(t,x_1,u_1) \right\| \leqslant l \left\{ \|x_2 - x_1\| + \|u_2 - u_1\| \right\}, \quad t \in [0,t_1]. \tag{1.3}$$

To this end, we prove that the linear part of this system

$$\begin{cases} w_{tt} + cw_t + dw_{xxxx} + kw^+ = u(t,x), \quad 0 < x < 1, \\ w(t,0) = w(t,1) = w_{xx}(t,0) = w_{xx}(t,1) = 0, \quad t \in \mathbb{R}, \end{cases} \tag{1.4}$$

is exactly controllable on $[0,t_1]$ for all $t_1 > 0$; moreover, we find the formula (4.7) to compute explicitly the control $u \in L^2(0,t_1; L^2(0,1))$ steering an initial state $z_0 = [w_0, v_0]^T$ to a final state $z_1 = [w_1, v_1]^T$ in time $t_1 > 0$ for the linear system (1.4). Then, we use this formula to construct a sequence of controls u_n that converges to a control u that steers an initial state z_0 to a final state z_1 for the non-linear system (1.2), which proves the exact controllability of this system. That is to say, the controllability of the linear system (1.4) is preserved under the non-linear perturbation $-kw^+ + p(t,x) + f(t,w,u(t,x))$.

2. Abstract formulation of the problem

The system (1.2) can be written as an abstract second order equation on the Hilbert space $X = L^2(0,1)$ as follows:

$$\ddot{w} + c\dot{w} + dAw + kw^+ = P(t) + u(t) + f\big(t,w,u(t)\big), \quad t \in \mathbb{R}, \tag{2.1}$$

where the unbounded operator A is given by $A\phi = \phi_{xxxx}$ with domain $D(A) = \{\phi \in X$: $\phi, \phi_x, \phi_{xx}, \phi_{xxx}$ are absolutely continuous, $\phi_{xxxx} \in X$; $\phi(0) = \phi(1) = \phi_{xx}(0) = \phi_{xx}(1) = 1\}$, and has the following spectral decomposition: