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Fréchet differentiability of the solutions of a semilinear abstract Cauchy problem

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Abstract

Sufficient conditions are found under which the solutions $z(t;q)$ of a semilinear abstract Cauchy problem of the form $\frac{d}{dt}z(t) = A(q)z(t) + F(q,t,z(t))$ are Fréchet differentiable with respect to the parameter q. An explicit form is provided for the sensitivity equation satisfied by the Fréchet derivative $D_q z(t;q)$.

Keywords: Abstract Cauchy problem; Analytic semigroup; Infinitesimal generator; Fréchet differentiability; Fréchet derivative

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1. Introduction

In this article we consider the problem of dependence on an unknown parameter q of the solution $z(t;q)$ of the semilinear abstract Cauchy problem

$$(\mathcal{P})_q \quad \begin{cases} \frac{d}{dt}z(t) = A(q)z(t) + F(q,t,z(t)), & z(t) \in Z, \\ z(0) = z_0, & t \in [0,T], \end{cases}$$

where Z, a Banach space, $q \in Q_{\text{ad}} \subset Q$, a normed linear space (Q_{ad} is an open subset of Q), and $A(q)$ is the infinitesimal generator of an analytic semigroup $T(t;q)$ on Z for all $q \in Q_{\text{ad}}$. The spaces Z and Q are referred to as the state space and the parameter space, respectively, while Q_{ad} will be referred to as the admissible parameter set. The set Q_{ad} reflects the fact that sometimes not all elements of Q are "admissible" for the particular problem at hand, although quite often one has $Q_{\text{ad}} = Q$.

Parameter identification problems for system $(\mathcal{P})_q$ and other similar type of equations [2,5,7] are usually solved by direct methods such as quasilinearization. For the application of these methods it is essential that solutions be differentiable with respect to the parameter q. For a concrete application of quasilinearization in a model similar to $(\mathcal{P})_q$ for the dynamics of Shape Memory Alloys see [10].

In 1977, Clark and Gibson [4] analyzed the differentiability of solutions in linear abstract Cauchy problems of the type

$$\frac{d}{dt}z(t) = A(q)z(t) + u(t),$$

where $A(q)$ generates a strongly continuous semigroup and $A(q) = A + B(q)$ where $B(q)$ is assumed to be bounded. That is, the dependence on q comes through a bounded component of $A(q)$.

Later on, in 1982 [1] this problem was studied under weaker assumptions, allowing for the parameter q to appear in unbounded terms of $A(q)$.

In 2000 Burns et al. [3] derived conditions under which the solutions of nonlinear Cauchy problems of the type

$$\frac{d}{dt}z(t) = Az(t) + F\big(q,t,z(t)\big),$$

are differentiable with respect to the parameter q. In this case, the parameter q was not allowed to appear in the linear part of the equation.

In this article we shall obtain conditions under which the solutions of the general abstract Cauchy problem $(\mathcal{P})_q$ are Fréchet differentiable with respect to q. To our knowledge, this problem has never been dealt with before. Moreover, we will prove that, under certain conditions, the corresponding Fréchet derivatives are solutions of particular nonhomogeneous evolution equations called the "sensitivity equations." We will provide an explicit form for these equations.

2. Preliminary results

Throughout this paper we shall consider the following standing hypotheses: