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Time-dependent potential scattering in chiral media

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Abstract

Electromagnetic waves propagating in a homogeneous three-dimensional unbounded chiral medium are considered. We define a chiral operator and study potential scattering relative to this operator. A spectral analysis of associated operators is obtained, based on the Plancherel theory of the Fourier transform. Using the generalised eigenfunction expansion theory, we give an integral representation of the solution. A discussion of asymptotic equality of solutions is provided and the associated wave operator introduced.

Keywords: Potential scattering; Spectral analysis; Chiral media

1. Introduction

Chiral materials are those which exhibit optical activity in the sense that the plane of vibration of linearly polarized light is rotated on passage through the material. Consequently, chiral phenomena in a medium can be investigated analytically by introducing into the classical Maxwell's equations those constitutive relations indicating the coupling of the electric and the magnetic fields which involve a so-called chirality measure. There are a number

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of such relations and note we shall use the Drude–Born–Fedorov [15,16], constitutive relations which are symmetric under time reversality and duality transformations.

In recent years chiral materials have been increasingly studied and there is a growing literature covering both their applications and the theoretical investigation of their properties. In this connection we would cite, in particular, the books [13–15,18] and the articles [1–3, 9,10,27], which give a comprehensive account of research activities centred on chiral materials. It will be noticed that the works dealing with wave phenomena in chiral materials have been mainly concerned with the study of time-harmonic waves which leads to frequency domain studies. The literature concerned with time domain studies in chiral media is, as yet, not very extended. We would refer to [4,7,12,19], which are representative of the work in this area. For a time domain analysis in achiral media we refer to the books [6,21, 30] and to the articles [22–26,28,29] (see also the books [5,8,17,20]).

In Section 2 we begin the formulation of the scattering problems of interest by introducing the equations governing electromagnetic wave motions in chiral media. We show that subject to an assumption on either the electric or the magnetic fields these governing equations can be reduced, in each case to the same generic form. We also introduce the notion of a chiral operator. In Section 3 we introduce the concepts of free and perturbed problems relative to the chiral operator. A spectral analysis of operators associated with a related potential problem is developed in Section 4 using the Plancherel theory of Fourier transforms. Finally, in Section 5 we briefly mention the asymptotic equality of solutions and define wave operators.

2. Formulation of the problem

We consider electromagnetic waves propagating in a homogeneous three-dimensional unbounded chiral medium. The electric field $E(x,t)$ and the magnetic field $H(x,t)$ satisfy the Maxwell's equations

$$\operatorname{curl} E(x,t) = -\frac{\partial B}{\partial t}(x,t), \tag{2.1}$$

$$\operatorname{curl} H(x,t) = \frac{\partial D}{\partial t}(x,t), \tag{2.2}$$

$$\operatorname{div} B(x,t) = 0, \qquad \operatorname{div} D(x,t) = 0, \tag{2.3}$$

where $D(x,t)$ and $B(x,t)$ are the electric and magnetic flux densities, respectively. We shall introduce constitutive relations which relate the material fields $H(x,t)$ and $D(x,t)$ to the primitive fields $E(x,t)$ and $B(x,t)$. Commonly used constitutive equations when investigating chiral effects are the Drude–Born–Fedorov (DBF) relations [14,15]

$$D(x,t) = \varepsilon\{I + \beta \operatorname{curl}\}E(x,t), \tag{2.4}$$

$$B(x,t) = \mu\{I + \beta \operatorname{curl}\}H(x,t), \tag{2.5}$$

where I is the identity operator and ε denotes the electric permittivity, μ the magnetic permeability and β the chirality measure. It is clear that if the chirality measure is equal to zero, then (2.4) and (2.5) reduce to the classical constitutive equations for the achiral case.