Available online at www.sciencedirect.com
SCIENCE @ DIRECT®

ELSEVIER

J. Math. Anal. Appl. 310 (2005) 81–96

Journal of MATHEMATICAL ANALYSIS AND APPLICATIONS

www.elsevier.com/locate/jmaa

The existence of almost periodic solutions of certain perturbation systems ☆

Yonghui Xia [a,b], Muren Lin [a], Jinde Cao [b,*]

[a] *College of Mathematics and Computer Science, Fuzhou University, Fuzhou, Fujian 350002, China*
[b] *Department of Mathematics, Southeast University, Nanjing 210096, China*

Received 3 July 2004

Available online 8 March 2005

Submitted by J.S.W. Wong

Abstract

Certain almost periodic perturbation systems are considered in this paper. By using the roughness theory of exponential dichotomies and the contraction mapping principle, some sufficient conditions are obtained for the existence and uniqueness of almost periodic solution of the above systems. Our results generalize those in [J.K. Hale, Ordinary Differential Equations, Krieger, Huntington, 1980; C. He, Existence of almost periodic solutions of perturbation systems, Ann. Differential Equations 9 (1992) 173–181; M. Lin, The existence of almost periodic solution and bounded solution of perturbation systems, Acta Math. Sinica 22A (2002) 61–70 (in Chinese); W.A. Coppel, Almost periodic properties of ordinary differential equations, Ann. Math. Pura Appl. 76 (1967) 27–50; A.M. Fink, Almost Periodic Differential Equations, Lecture Notes in Math., vol. 377, Springer-Verlag, New York, 1974; Y. Xia, F. Chen, A. Chen, J. Cao, Existence and global attractivity of an almost periodic ecological model, Appl. Math. Comput. 157 (2004) 449–475].

Keywords: Contraction mapping; Exponential dichotomies; Roughness; Almost periodic solution

☆ This work was supported by the National Natural Science Foundation of China under Grant 60373067, the Natural Science Foundation of Jiangsu Province, China under Grant BK2003053, Qing-Lan Engineering Project of Jiangsu Province, China, the Foundation of Developing Science and Technology of Fuzhou University under the grant 0030824594, and the Foundation of Fujian Education Bureau Under the Grant JB03059.

* Corresponding author.
E-mail addresses: yhxia@fzu.edu.cn, xiayonhui@163.com (Y. Xia), jdcao@seu.edu.cn (J. Cao).

0022-247X/$ – see front matter
doi:10.1016/j.jmaa.2005.01.046

1. Introduction

In [1], Hale considered the following periodic systems containing a small parameter of the form:

$$\frac{dx}{dt} = Ax + \varepsilon g(t,x), \qquad \frac{dx}{dt} = A(t)x + \varepsilon g(t,x), \qquad \frac{dx}{dt} = A(t)x + g(t,x,\varepsilon).$$

Some sufficient conditions were obtained for the existence of ω-periodic solution of the above systems. However, if the various constituent components of the temporally non-uniform environment is with incommensurable (non-integral multiples) periods, then one has to consider the environment to be almost periodic since there is no a priori reason to expect the existence of periodic solutions. If we consider the effects of the environmental factors, the assumption of almost periodicity is more realistic, more important and more general. In 1974 Fink [8] investigated the perturbation system

$$\frac{dx}{dt} = A(t)x + \varepsilon g(t,x,\varepsilon). \tag{1.1}$$

Some sufficient conditions were obtained for the existence and uniqueness of almost periodic solution of system (1.1). Then Lin [2] and He [3] considered system

$$\frac{dx}{dt} = A(t)x + f(t) + \varepsilon g(t,x,\varepsilon). \tag{1.2}$$

By using the contraction mapping principle and exponential dichotomies theory, some sufficient conditions were obtained for the existence and uniqueness of almost periodic solution of (1.2), respectively. Recently, Lin [4] generalized system (1.2) to perturbation system:

$$\frac{dx}{dt} = A(t,\varepsilon)x + f(t) + \varepsilon g(t,x,\varepsilon). \tag{1.3}$$

By using the contraction mapping principle and exponential dichotomies theory, some sufficient conditions were obtained for the existence and uniqueness of almost periodic solution and bounded solutions of (1.3), respectively.

In this paper, we consider the following systems

$$\frac{dx}{dt} = A(t)x + f(t,x) + \varepsilon g(t,x,\varepsilon) \quad \text{and} \tag{1.4}$$

$$\frac{dx}{dt} = A(t,\varepsilon)x + f(t,x) + \varepsilon g(t,x,\varepsilon). \tag{1.5}$$

Motivated by the works [1,3–8], we combine the roughness theory with the contraction mapping principle to study the above systems. Some sufficient conditions are obtained for the existence and uniqueness of almost periodic solutions of the above systems. To the best of authors' knowledge, this is the first paper considering systems (1.4) and (1.5) by this method. Moreover, because we extend $f(t)$ in system (1.2) or system (1.3) to $f(t,x)$, the method by using the contraction mapping principle and exponential dichotomies theory only cannot be applied to system (1.4) and (1.5). Then the roughness theory must be employed to study system (1.4) and (1.5).