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Positive solutions of a system of non-autonomous fractional differential equations

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Abstract

Existence of positive solutions for the following system of fractional differential equations:

$$D^{\alpha_i} u_i = f_i(t, u_1, u_2, \ldots, u_n), \quad u_i(0) = 0, \quad 0 < \alpha_i < 1,\ 1 \leqslant i \leqslant n,$$

where D^{α_i} denotes Riemann–Liouville derivative of order α_i has been studied.

Keywords: Riemann–Liouville fractional derivative; Completely continuous operator; Arzela–Ascoli theorem; Ordered Banach space

1. Introduction

Fractional differential equations have received considerable attention in the recent literature [9,10]. Atanackovic and Stankovic [1] have analysed lateral motion of an elastic column fixed at one end and loaded at the other in terms of a system of fractional differential equations. Daftardar-Gejji and Babakhani [3] have presented analysis of a system of fractional differential equations. Numerous applications of fractional differential equations in different areas of physics, engineering and biological sciences have been presented in the recent literature [4,8–12]. Earlier, Zhang [13] has explored the differential equation:

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$D^{\alpha}u(t) = f(t,u)$, $0 < \alpha < 1$, and investigated existence of positive solutions. Babakhani and Daftardar-Gejji [2] have presented the detailed analysis of the existence of positive solutions of multi-term differential equation: $L(D)u = f(x,u)$, where

$$L(D) = D^{s_n} - a_{n-1}D^{s_{n-1}} - \cdots - a_1 D^{s_1}, \quad 0 < s_1 < \cdots < s_n < 1,\ a_j > 0.$$

As a pursuit of this in the present paper we deal with the existence of positive solutions for the system of fractional differential equations given by

$$D^{\alpha_i}u_i = f_i\,(t, u_1, \ldots, u_n), \quad 0 < \alpha_i < 1,\ 1 \leqslant i \leqslant n.$$

The paper has been organized as follows. In Section 2, basic definitions and properties related to fractional derivatives and integrals have been summarized. This has been followed by the main results proved in Section 3.

2. Preliminaries

We give some basic definitions [7–9] and theorems which are used further in this paper.

Definition 2.1. Let $f:[a,b] \to R$, and $f \in L^1[a,b]$. The left-sided Riemann–Liouville fractional integral of f of order α is defined as

$$I_a^{\alpha} f(x) = \frac{1}{\Gamma(\alpha)} \int_a^x \frac{f(t)}{(x-t)^{(1-\alpha)}}\,dt, \tag{1}$$

where $\alpha > 0$, $a < x < b$.

Definition 2.2. The left-sided Riemann–Liouville fractional derivative of a function $f:[a,b] \to R$ is defined as

$$D_a^{\alpha} f(x) = D^m I_a^{m-\alpha} f(x), \tag{2}$$

where $m = [\alpha] + 1$, $D^m = \frac{d^m}{dt^m}$, $a < x < b$.

Hereafter D^{α} denotes D_0^{α} and I^{α} denotes I_0^{α}. If the fractional derivative $D_a^{\alpha} f(t)$ is integrable, then [9]

$$I_a^{\alpha}\big(D_a^{\beta} f(t)\big) = I_a^{\alpha-\beta} f(t) - \big[I_a^{1-\beta} f(t)\big]_{t=a} \frac{(t-a)^{\alpha-1}}{\Gamma(\alpha)}, \quad 0 < \beta \leqslant \alpha < 1. \tag{3}$$

If $f \in C[a,b]$, then $[I_a^{1-\beta} f(t)]_{t=a} = 0$, and Eq. (3) takes the form

$$I_a^{\alpha}\big(D_a^{\beta} f(x)\big) = I_a^{\alpha-\beta}, \quad 0 < \beta \leqslant \alpha < 1. \tag{4}$$

Definition 2.3 [5]. A Banach space $\mathcal{B}$ endowed with a closed cone K is an ordered Banach space $(\mathcal{B}, K)$ with a partial order $\leqslant$ in $\mathcal{B}$ as follows:

$$x \leqslant y \quad \text{if } y - x \in K.$$