Available online at www.sciencedirect.com
SCIENCE DIRECT®

J. Math. Anal. Appl. 302 (2005) 97–128

Journal of MATHEMATICAL ANALYSIS AND APPLICATIONS
www.elsevier.com/locate/jmaa

Quasiasymptotic analysis in Colombeau algebra

Stevan Pilipović, Mirjana Stojanović *

Department of Mathematics and Informatics, Faculty of Science, University of Novi Sad, Trg D. Obradovića 4, 21 000 Novi Sad, Serbia and Montenegro

Received 20 June 2002

Submitted by S. Kaijser

Abstract

We consider the main notions of $\mathcal{G}$-quasiasymptotic ($\mathcal{G}$-q.a.) analysis: $\mathcal{G}$-q.a. behavior at zero, $\mathcal{G}$-q.a. expansion at infinity, *Sc*-asymptotic and boundedness, *Sc*-expansion at infinity and their applications to the asymptotic behavior of the solution to some types of nonlinear PDEs in Colombeau algebra of generalized functions $\mathcal{G}$.
© 2004 Elsevier Inc. All rights reserved.

Keywords: Colombeau algebra of generalized functions; $\mathcal{G}$-q.a. behavior at zero; Semilinear hyperbolic system; $\mathcal{G}$-q.a. expansion at infinity; Wave equation; *Sc*-asymptotic; *Sc*-boundedness; Semilinear parabolic equations; *Sc*-asymptotic expansion

1. Introduction

The q.a. behavior [27] and S-asymptotic [24] play important role in the investigations of the asymptotic behavior of the generalized functions' integral transforms. Also, asymptotic behaviors of distributions were used in investigations of analytical properties of quantum field elements. These results pushed forward the study of analytical properties of distributions and so the mathematical tool was developed (cf. [5,11,25,27]). Notions of q.a. and

* Corresponding author.
E-mail addresses: pilipovic@im.ns.ac.yu (S. Pilipović), stojanovic@im.ns.ac.yu, stojanovicm@yahoo.com (M. Stojanović).

0022-247X/$ – see front matter © 2004 Elsevier Inc. All rights reserved.
doi:10.1016/j.jmaa.2004.07.044

S-asymptotic behavior and expansions of Schwartz distributions are analyzed in [25,27] (cf. references of these monographs). Systematic and simplified approach to asymptotic expansions by the use of distributions is given in [8].

Distributions in the framework of asymptotic analysis allow preformation of many analytical operations and assign values of divergent integrals. These notions are extended to Colombeau algebra by the authors in [18,21,22]. Definition of $\mathcal{G}$-q.a. expansion at zero, basic properties and evaluation of some generalized functions in $\mathcal{G} \setminus \mathcal{D}'$ are given in [21] and [22]. We refer to [19] for related questions via $\mathcal{G}$-q.a. behavior at zero.

Asymptotic analysis in Colombeau algebra of generalized functions $\mathcal{G}$ which is the larger space than the space of distributions $\mathcal{D}'$ allowing their multiplications, has a lot of advantages. We prove that the properties of S-asymptotic and boundedness, q.a. behavior and expansion in $\mathcal{D}'$ are preserved in $\mathcal{G}$, but the space $\mathcal{G}$ is a source of new properties, useful for application. It preserves the classical properties but it gives new ones not available in classical approach. We introduced and investigated in [17] and [20] the $\mathcal{G}$-q.a. behavior at zero in $\mathcal{G}$. We give a new way of q.a. analysis and expansion in Colombeau setting outside the classical approach since we consider elements from $\mathcal{G} \setminus \mathcal{D}'$.

In this paper we consider the three main notions in q.a. analysis: q.a. behavior at zero, q.a. expansion and S-asymptotic at infinity in $\mathcal{G}$. We give the systematical extension of the q.a. method from Schwartz theory of distributions to Colombeau algebra $\mathcal{G}$. The applications of $\mathcal{G}$-q.a. behavior and *Sc*-asymptotic and boundedness to PDEs are displayed.

We show that the Cauchy problem for a semilinear strictly hyperbolic $(n \times n)$-system (2) in two independent variables, has a solution whose $\mathcal{G}$-q.a. behavior at zero is determined by the $\mathcal{G}$-q.a. behavior at zero of the initial data.

We give the definition of $\mathcal{G}$-q.a. expansion at infinity in Colombeau space $\mathcal{G}_t$ and describe its basic properties. We evaluate powers of δ^n, $n \in \mathbf{N}$. We expanded quasiasymptotically the $\delta^2(x,t)$ with respect (w.r.) to the corresponding scale and apply it in the analysis, of the singular part of wave equation of the form (20) below. We obtain the $\mathcal{G}$-q.a. expansion of the solution along the characteristic lines which is a consequence of $\mathcal{G}$-q.a. expansion of the initial data which are (δ^2, δ^2).

In spite of the fact that the embedding of distributions is not canonical in simplified version of Colombeau algebra that version is in the line with other Colombeau algebras. The nonuniqueness of the embedding is not obstacle for the application of the theory.

We give the extension of the notion of S-asymptotic and boundedness from $\mathcal{S}'$ to $\mathcal{G}_t$, as the notions of *Sc*-asymptotic, respectively *Sc*-boundedness at infinity in Colombeau algebra of tempered generalized functions as the analogous to the corresponding ones in classical theory. We give the main characterizations of these notions and application of them in solving semilinear parabolic equations with singular initial data. Results concerning this subject are given mostly in one space dimension. In many dimensional case instead of interval $[0,\infty)$, a cone $\mathbf{R}^n_+$ is used, and in the case of general cone Γ, the assertions are slightly complicated (cf. [25]). We analyze a semilinear parabolic equation with conservative nonlinear term (or without it) and singular initial data through the *Sc*-asymptotic, respectively *Sc*-boundedness of the initial data. In appropriate cases the *Sc*-asymptotic, respectively *Sc*-boundedness of the solution is obtained.

Download English Version:

https://daneshyari.com/en/article/9503042

Download Persian Version:

https://daneshyari.com/article/9503042

Daneshyari.com