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Applied Mathematics and Computation 171 (2005) 315–319
APPLIED MATHEMATICS AND COMPUTATION
www.elsevier.com/locate/amc

Explicit solutions of the generalized KdV equations with higher order nonlinearity

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Abstract

By constructing auxiliary functions and equations, we find some new explicit solutions of the generalized KdV equations with higher order nonlinearity. In particular, we get the peaked solitary wave solutions of the generalized KdV equations.

Keywords: The generalized KdV equations; Peaked solitary wave; Explicit solutions

For the generalized KdV equations (GKdVE) with higher order nonlinearity

$$u_t + a(1 + bu^n)u^n u_x + \delta u_{xxx} = 0. \tag{1}$$

Dey [1,2] discussed the domain wall solutions and Hamiltonians of GKdVE (1), where a, b, n and $\delta(>0)$ are given constants. In [3], the solitary waves and

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0096-3003/$ - see front matter © 2005 Published by Elsevier Inc.
doi:10.1016/j.amc.2005.01.053

bifurcations of GKdVE (1) gave been studied for $\delta = 1$. When $n = 1, 2$, GKdVE (1) becomes the well-known KdV or mKdV equations.

In this letter, by constructing auxiliary functions and equations, we shall find new solitary waves solutions of GKdVE (1) with n being an arbitrary positive constant.

For the sake of simplicity, we take that $\delta = 1$ in this paper. To look for solitary waves solutions of GKdVE (1), lets take that $u = u(\zeta)$ and $\zeta = x - ct$, where c is a constant to be determined. Substituting this transformation into GKdVE (1) and integrating once with respect to ζ, yields

$$u'' + \frac{ab}{2n+1}u^{2n+1} + \frac{a}{n+1}u^{n+1} - cu = 0, \tag{2}$$

where $u' = \mathrm{d}u/\mathrm{d}\zeta$. For Eq. (2), one can prove the following result

Proposition 1. *If the function* $u(\zeta)$ *is a solution of Eq.* (2), *then the functions* $u(-\zeta)$ *and* $u(\pm|\zeta|)$ *are also solutions of Eq.* (2).

The result of Proposition 1 denotes the there exist peaked solitary wave solution of GKdVE (1). This peaked solitary wave solution, by Camassa and Holm, was presented for the so-called Camassa–Holm shallow water wave equation [4,5].

Now we look for the explicit solutions of Eq. (2). To this end, constructing the following auxiliary functions

$$u^n(\zeta) = w(\zeta) - r, \tag{3}$$

where r is a constant to be defined. It follows from (2) and (3) that

$$\frac{w-r}{n}w'' - \frac{n-1}{n^2}w'^2 + \frac{ab}{2n+1}(w-r)^4 + \frac{a}{n+1}(w-r)^3 - c(w-r)^2 = 0. \tag{4}$$

To get explicit solutions of Eq. (4), we divide the following two cases.

Case 1: $r = 0$ For this case, introducing the auxiliary equation

$$w'^2 = pw^4 + mw^3 + qw^2, \tag{5}$$

where p, m and q are constants to be defined. Substituting (5) into Eq. (4), one can get

$$\theta_4 w^4 + \theta_3 w^3 + \theta_2 w^2 = 0 \tag{6}$$

and

$$\theta_4 = p + \frac{p}{n} + \frac{abn}{2n+1}, \quad \theta_3 = \frac{m}{2n} + \frac{m}{n^2} + \frac{a}{n+1}, \quad \theta_2 = \frac{q}{n} - nc. \tag{7}$$