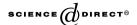


Available online at www.sciencedirect.com





ELSEVIER Applied Mathematics and Computation 171 (2005) 853–856

www.elsevier.com/locate/amc

On the system of high order rational difference equations $x_n = \frac{a}{y_{n-p}}$, $y_n = \frac{by_{n-p}}{x_{n-q}y_{n-q}}$

Xiaofan Yang *, Yaoxin Liu, Sen Bai

College of Computer Science, Chongqing University, Chongqing 400044, People's Republic of China

Abstract

In this paper, we study the behavior of positive solutions of the system of high order rational difference equations

$$x_n = \frac{a}{y_{n-p}}, \quad y_n = \frac{by_{n-p}}{x_{n-q}y_{n-q}}, \quad n = 1, 2, \dots,$$

where p and q are positive integers with $p \le q$, and a and b are positive constants. © 2005 Elsevier Inc. All rights reserved.

Keywords: Difference equation; Periodicity; Monotonicity; Recurrence relation

1. Introduction

In recent years, the behavior of positive solutions of various systems of rational difference equations has been one of the main topics in the theory of

^{*} Corresponding author.

E-mail addresses: xf_yang1964@yahoo.com, xf_yang@163.com, yxf640126@sina.com (X. Yang).

difference equations [1–5]. In particular, Cinar [1] proved that all positive solutions of the system of difference equations

$$x_n = \frac{1}{y_{n-1}}, \quad y_n = \frac{y_{n-1}}{x_{n-2}y_{n-2}}, \quad n = 1, 2, \dots$$
 (1.1)

are period four.

In this paper, we study the system of high order difference equations

$$x_n = \frac{a}{y_{n-p}}, \quad y_n = \frac{by_{n-p}}{x_{n-q}y_{n-q}}, \quad n = 1, 2, \dots,$$
 (1.2)

where p and q are positive integers with $p \le q$, and a and b are positive constants. When p = 1, q = 2, and a = b = 1, Eq. (1.2) reduces to Eq. (1.1).

2. Main results

We first examine the periodicity of positive solutions of Eq. (1.2).

Theorem 2.1. If a = b, and q is divisible by p, then every positive solution of Eq. (1.2) is period 2q.

Proof. Assume q = rp, where r is a positive integer. Consider an arbitrary positive solution $\{(x_n, y_n)\}_{n=-(q-1)}^{\infty}$ of Eq. (1.2). For each $n \ge q+1$, replacing $x_{n-q} = \frac{a}{y_{n-(p+q)}}$ into $y_n = \frac{by_{n-p}}{x_{n-q}y_{n-q}}$ and multiplying both sides by y_{n-q} , we get the recurrence relation

$$y_n y_{n-q} = y_{n-p} y_{n-(p+q)}. (2.1)$$

It follows by repeatedly applying this recurrence relation that

$$y_n y_{n-q} = y_{n-p} y_{n-(p+q)} = y_{n-2p} y_{n-(2p+q)} = \dots = y_{n-rp} y_{n-(rp+q)} = y_{n-q} y_{n-2q}$$

So $y_n = y_{n-2q}$ and hence $x_n = \frac{a}{y_{n-p}} = \frac{a}{y_{n-p-2q}} = x_{n-2q}$. The proof is completed. \square

As a direct consequence of this theorem, we again obtain the result in [1] as follows.

Corollary 2.2. Every positive solution of Eq. (1.1) is period 4.

We then examine the monotonicity of positive solutions of Eq. (1.2).

Theorem 2.3. Assume q is divisible by p. Let $\{(x_n, y_n)\}_{n=-(q-1)}^{\infty}$ be an arbitrary positive solution of Eq. (1.2).

Download English Version:

https://daneshyari.com/en/article/9506249

Download Persian Version:

https://daneshyari.com/article/9506249

Daneshyari.com