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A numerical study for computation of geodesic curves

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Abstract

Consideration is given to the numerical computation of geodesic on surfaces. The method of finite-difference is used for governing non-linear system of differential equations. Then the resulting non-linear algebraic equations are solved by both iterative and Newton's method. It is shown that iterative method (IM) gives better result than Newton's method. Finally, we demonstrated our finding on several surfaces.

Keywords: Geodesic; Finite-difference; Relaxation methods

1. Introduction

In an axiomatic approach to geometry we study the properties of points and lines. Most of the theorems in axiomatic geometry deal with the relationships between points and lines. If we are to see how the differential geometry we have

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been studying is to relate to axiomatic geometry, we need some method for developing an abstract definition of a line. This is different from our axiomatic technique of taking a line as an undefined term. There are various ways in which a straight line in usual Euclidean geometry can be characterized. For instance, it has zero curvature everywhere, all its tangent vectors are parallel, or it is the solution of the simple first order linear differential equation $\ddot{v}(t) = V_0$. Neither of these characterizations can be immediately transferred to the case of curves within a Riemannian manifold but the following definition is generalizable: A straight line between two points is the curve which minimizes the distance between these points. Since in a Riemannian metric we have the notion of "length", we can use this to define what a "straight line" in a curved space is. Such "straight lines" are called "geodesics". Geometrically, a geodesic on a surface is an embedded simple curve on the surface such that for any two points on the curve the portion of the curve connecting them is also the shortest path between them on the surface.

A different characterization of a geodesic is the following: A curve on a surface is geodesic if and only if the normal vector to the curve is everywhere parallel to the local normal vector of the surface. This goes back to Johann Bernoulli (1697)! Yet another way to characterize a geodesic is the following: Geodesics are curves along which geodesic curvature K_{g} vanishes. This is of course where the geodesic curvature has its name from. The geodesic curvature measures the projection of the curvature of a curve onto the local tangent plane, i.e., the part which is in some sense independent of the curvature of the surface. Hence, requiring $K_{\mathrm{g}} = 0$ means, loosely speaking, geodesics have no curvature other than the inevitable curvature which is due to the bending of the surface itself.

Geodesic on a surface is an intrinsic geometric feature that plays an important role in a diversity of applications. Many geometric operations are inherently related to geodesics. For instance, when a developable surface is flattened into a planer figure (with no distortion), any geodesic on it will be mapped to a straight line in the planer figure [1]. Thus, to flatten an arbitrary non-developable surface with as little distortion as possible, a good algorithm should try to preserve the geodesic curvatures on the surface [2,3]. Geodesic method also finds its applications in computer vision and image processing, such as in object segmentation [4–6] and multi-scale image analysis [7,8]. The concept of geodesic also finds its place in various industrial applications, such as tent manufacturing, cutting and painting path, fiberglas tape windings in pipe manufacturing, textile manufacturing [9–16].

Traditional fundamental research in geodesics contentrated on finding and characterizing geodesics on analytical curved surfaces [17]. As the computer becomes increasingly more powerful, and discretized models become more prevalent in geometric modeling, discrete geodesics have also been gaining attention.