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Cubic spline for a class of singular two-point boundary value problems

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Abstract

In this paper we have presented a method based on cubic splines for solving a class of singular two-point boundary value problems. The original differential equation is modified at the singular point then the boundary value problem is treated by using cubic spline approximation. The tridiagonal system resulting from the spline approximation is efficiently solved by Thomas algorithm. Some model problems are solved, and the numerical results are compared with exact solution.

Keywords: Ordinary differential equations; Singular point; Two-point boundary value problem; Cubic spline

1. Introduction

Many problems in applied mathematics leads to singular boundary value problems of the form:

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$$y''(x) + \frac{k}{x}y'(x) + b(x)y(x) = c(x), \quad 0 < x < 1,$$

subject to the boundary conditions $y'(0) = 0$, and $y(1) = \beta$. Which arise in the study of generalized axially symmetric potentials after separation of variables has been employed. These problems also occur very frequently in the study of electrohydrodynamics and the theory of thermal explosions. Singular boundary value problems have been studied by several authors [4,6,7,9]. Albasiny and Hoskins [2] have obtained spline solutions by solving a set of equations with a tridiagonal matrix of coefficients. Bickely [3] has considered the use of cubic spline for solving linear two point boundary value problems. Later, Fyfe [5] discussed the application of deferred corrections to the method suggested by Bickley, by considering linear boundary value problems. In comparison with the finite difference methods, spline solution has its own advantages. For example, once the solution has been computed the information required for spline interpolation between mesh points is available. This is particularly important when the solution of boundary value problem is required at various locations in the interval $[a, b]$.

Attempts by many researchers for the removal of singularity are based on using the series expansion procedures in the neighborhood $(0, \delta)$ of singularity and then solve the regular boundary value problem in the interval $(\delta, 1)$ using any numerical method. In this paper, we discuss a direct method based on cubic splines for a class of singular two-point boundary value problems. The original differential equation is modified at the singular point. The cubic approximation is then employed to solve the boundary value problem. The cubic spline approximation leads to the tridiagonal system of equations, which is solved by Thomas algorithm. Some numerical examples have been solved to demonstrate the efficiency of the method.

2. Description of the method

Consider a class of singular two-point boundary value problems of the form:

$$y''(x) + \frac{k}{x}y'(x) + b(x)y(x) = c(x), \quad 0 < x < 1 \tag{2.1}$$

the parameter $k \geqslant 1$ with the boundary conditions

$$y'(0) = 0, \quad y(1) = \beta. \tag{2.1a, b}$$

Since $x = 0$ is singular point of Eq. (2.1), we first modify the Eq. (2.1) at $x = 0$. By L'Hospital rule, the boundary value problem

$$y''(x) + \frac{k}{x}y'(x) + b(x)y(x) = c(x),$$
$$y'(0) = 0, \quad y(1) = \beta$$