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Ratio methods to the mean estimation with known quantiles

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Abstract

We address the problem of estimating the finite population mean in survey sampling, by exploiting any available auxiliary information in order to increase the precision of classical estimators. The idea is to use any population quantiles of the available auxiliary variables which are known in many real situation from census, administrative files, etc. This is achieved using these known quantities in the construction of the estimators, by modifying the usual ratio estimation methods and afterwards defining a general class of exponentiation ratio estimators. The advantages of the proposed estimators are demonstrated using theoretical asymptotic tools and through a simulation study.

Keywords: Auxiliary information; Exponentiation methods; Quantiles; Ratio estimator

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1. Introduction

In sample surveys, auxiliary information on the finite population is often available, i.e. some population characteristics of an auxiliary variable x are known. By using the relationships between the study variable y, and the auxiliary variable x, the estimate of a parameter $\theta(y)$ can be improved. The most commonly available auxiliary information is knowledge of $\overline{X}$, in which case $\overline{Y}$ is often estimated by the ratio estimator.

Other methods include the regression, difference, generalized regression (GREG), calibrated and empirical likelihood estimators. A limitation of the methods mentioned above is that they used only a single auxiliary parameter (specifically the population mean) associated with the auxiliary variable.

The ratio estimator of population mean is extensively used in practice, specially with large-scale sample surveys containing many items, because its computational simplicity, applicability to general sampling designs, and increased efficiency through the utilization of concomitant information.

Rao [9], Mayor [4] and Singh [15] provided a review on ratio estimators. Ratio type estimators with multiple auxiliary variables are considered by Olkin [5], Shukla [13], Wrigth [20], Rao and Mudholkar [10], Agarwal et al. [1], Raj [6], ...

It is very common that the population data associated with the auxiliary variables are obtained from census, administrative files, etc. and these sources often provide different parameters of these auxiliary variables. For example, it is usual to provide some position measures (mean, median and other moments). Thus, we wish to determine how the knowledge of these various parameters associated with the auxiliary variable x, is used to propose ratio estimators more efficient than the standard ratio estimators that uses only the mean $\overline{X}$ as auxiliary information.

The use of a parameter of the auxiliary variable on estimating a different parameter of the main variable is not a new idea. For example, Srivastava and Jhajj [17] proposed a class of estimators of the mean when the mean and the variance are known. Singh and Mathur [16] considered the estimation of population mean when coefficient of variation is known. Upadhyaya and Singh [19] proposed two ratio-type estimators when the mean $\overline{X}$, the coefficient of variation C_x and the coefficient of kurtosis β_x, are known. Recently, Shabbir and Yaab [12] propose a class of ratio estimators using the known values $\overline{X}$, C_x and C_{xy}. These studies were developed under simple random sampling and do not extend to more complex survey designs.

In this work we present two ratio-type estimators to make efficient use of the knowledge of quantiles of several auxiliary variables for a general sampling design, d.