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Remark on convergence of algebraic multigrid in the form of matrix decomposition

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Abstract

We introduce the convergence of algebraic multigrid in the form of matrix decomposition. The convergence is proved in block versions of the multi-elimination incomplete LU (BILUM) factorization technique and the approximation of their inverses to preserve sparsity. The convergence theorem can be applied to general interpolation operator. Furthermore, we discuss the error caused by the error matrix.

Keywords: Algebraic multigrid; Matrix decomposition; Convergence

1. Introduction

In multigrid (MG) methods there are two basic steps: error smoothing and coarse-grid correction [3,4,11]. Geometric MG (GMG) methods employ fixed grid hierarchies and, therefore, an efficient interplay between smoothing and

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coarse-grid correction is ensured by selecting appropriate smoothing processes. In contrast to this, algebraic MG (AMG) methods attempt to maintain simple smoothers by employing reasonable operator-dependent interpolation and the Galerkin operator to achieve robust convergence [5–9,12]. Ruge and Stüben develop an efficient AMG interpolation operator for M-matrices [12]. Chang et al. present another interpolation formula based on the following geometric assumption: the size of the matrix entries is assumed to reflect the distance between grid points [8]. In [10], a new algorithm for computing interpolation weights is described. Chang and Huang improved the interpolation operators and gave a new convergence theorem of new AMG algorithms [6].

The multi-elimination ILU decomposition (ILUM), introduced in [14], is based on exploiting the idea of successive independent set ordering. Block versions of ILUM decomposition are attractive because the point ILUM factorization may have difficulties when the diagonal elements of the resulting U factor are small [2,14–17,19]. Basing on block versions of ILUM (BILUM), we get the approximate matrix decomposition

$$A = \begin{pmatrix} I & 0 \\ \widehat{ED^{-1}} & I \end{pmatrix} \begin{pmatrix} D & 0 \\ 0 & A_1 \end{pmatrix} \begin{pmatrix} I & \widehat{D^{-1}F} \\ 0 & I \end{pmatrix} + \begin{pmatrix} 0 & R_2 \\ R_0 & R_1 \end{pmatrix},$$

where D is a square matrix, $\widehat{ED^{-1}}$, $\widehat{D^{-1}F}$ are the approximate matrices of ED^{-1}, $D^{-1}F$, respectively, and

$$R = \begin{pmatrix} 0 & R_2 \\ R_0 & R_1 \end{pmatrix}$$

is the error matrix.

The paper is organized as follows. The relations between BILUM and AMG are discussed in Section 2. In Section 3, the convergence of the AMG methods in the form of matrix decomposition is proved and the theorem is proved that it can be used to special examples of Chang and Saad. Finally, we consider the quality of the error matrix in Section 4.

2. BILUM and AMG

A block independent set (BIS) is a set of groups (blocks) of unknowns such that there is no coupling between unknowns of any two different groups (blocks) [16]. Suppose that a (block) independent set ordering has been found by one of the techniques introduced in [14,16,18]. Then the original matrix A can be permuted into a 2×2 block matrix (A) of the form

$$A \sim PAP^{\mathrm{T}} \equiv \begin{pmatrix} D & F \\ E & C \end{pmatrix},$$