Available online at www.sciencedirect.com
SCIENCE DIRECT®
ELSEVIER Applied Mathematics and Computation 170 (2005) 1209–1242
APPLIED MATHEMATICS AND COMPUTATION
www.elsevier.com/locate/amc

Splitting as an approach to constructing local exact artificial boundary conditions

Denis M. Filatov

Centre for Atmospheric Science (CCA), National Autonomous University of Mexico (UNAM), Circuito Exterior, C.P. 04510, Mexico City, Mexico

Abstract

We employ the technique of splitting for constructing artificial boundary conditions (ABCs) for the linear advection–diffusion–reaction equation when the computational domain is an nD open set with a piecewise smooth artificial boundary. The splitting is performed both by the physical processes and by coordinates. The former permits to construct ABCs for each of the processes separately, which provides local exact boundary conditions; the latter leads to ABCs much less exigent to the shape of artificial boundary in comparison with many others. We also prove that the corresponding boundary value problems are well-posed, and present results of the numerical experiments that confirm the theoretical study.

Keywords: Local artificial boundary conditions; Advection–diffusion–reaction equation; Operator splitting; Spline approximation; Piecewise smooth artificial boundary

E-mail address: denisfilatov@mail.ru

0096-3003/$ - see front matter
doi:10.1016/j.amc.2005.01.012

1. Introduction

While numerically solving a differential problem originally formulated in an unbounded domain, one has to reformulate it for a finite computational domain and impose adequate boundary conditions on the artificial boundary. These conditions are often called artificial boundary conditions (ABCs), and they are to satisfy the two requirements: first, they must lead to a well-posed boundary value problem (BVP); second, the error between the solutions to the original Cauchy problem and the resulting BVP should be as small as possible in the computational domain; furthermore, if the error is equal to zero then the ABC is said to be exact.

There are a large number of papers on constructing ABCs for solving various problems of mathematical physics (see, e.g., [1–20], also [21]). It should be noted, however, that all the methods developed by the moment and described in the literature possess certain restrictions. Obviously, the main reason of this is that the methods inherit their disadvantages from the analytical techniques which they are based on. For instance, methods based on the use of the Fourier transform in space lead to ABCs that can be applied in case of a rectangular artificial boundary only; ABCs constructed with the employment of the apparatus of generalised solutions suffer from globality (either in time or in space, or even both), and hence, require to be somehow localised; methods that involve the concept of the Dirichlet-to-Neumann map produce ABCs expressed in terms of the normal derivative of the solution, and so, these boundary conditions fail when computing the solution at a singular point (i.e., at a "corner") of the boundary.

Among the papers quoted above we would like to emphasise the work by Gustafsson and Sundström [8], where the authors consider the compressible Navier–Stokes and viscous shallow-water equations. An interesting feature of [8] is that the ABCs are represented as a mixed Dirichlet–Neumann (or Robin) boundary condition, where the first term (the Dirichlet's) corresponds to the purely transport process, while the second one (the Neumann's) is responsible for the viscosity only. Such a representation can non-rigorously be treated as splitting by physical processes. Some issues on this topic are also analysed by Oliger and Sundström in [22], Section 5.

In this paper we investigate the technique of splitting by physical processes in its strict, classical sense [23] (see also [24,25]). This approach can become an efficient tool for constructing ABCs for multi-processed equations of mathematical physics. As the object of research, we choose the linear advection–diffusion–reaction equation, which is of great interest and importance in various practical applications, e.g., in environmental modelling [26–30]. Besides, we also study the splitting by coordinates (or dimensional splitting) [23–25] that allows essential reducing the restrictions on the shape of artificial boundary in comparison with many other ABC-constructing methods [21]. It should be