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Multiparametric sensitivity analysis of the constraint matrix in linear-plus-linear fractional programming problem

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Abstract

In this paper, we study multiparametric sensitivity analysis for programming problems with linear-plus-linear fractional objective function using the concept of maximum volume in the tolerance region. We construct critical regions for simultaneous and independent perturbations of one row or one column of the constraint matrix in the given problem. Necessary and sufficient conditions are given to classify perturbation parameters as 'focal' and 'nonfocal'. Nonfocal parameters can have unlimited variations, because of their low sensitivity in practice, these parameters can be deleted from the analysis. For focal parameters, a maximum volume tolerance region is characterized. Theoretical results are illustrated with the help of a numerical example.

Keywords: Multiparametric sensitivity analysis; Maximum volume region; Generalized fractional programming; Tolerance approach; Parametric programming

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1. Introduction

A general linear-plus-linear fractional programming problem has the following form:

$$
\begin{aligned}
\text{(LLFP)} \quad \text{Maximize} \quad & F(x) = c^{\mathrm{T}}x + \frac{p^{\mathrm{T}}x}{q^{\mathrm{T}}x + \theta} \\
\text{subject to} \quad & Ax = b, \\
& x \geqslant 0,
\end{aligned}
$$

where A is $m \times n$ coefficient matrix with $m < n$; c^{T}, p^{T} and q^{T} are n-dimensional row vectors; x and b are n-dimensional and m-dimensional column vectors respectively and θ is a scalar quantity. It is assumed that the feasible region of the problem (LLFP) is bounded.

Teterev [14] pointed out that such problems arise when a compromise between absolute and relative terms is to be maximized. Major applications of the problem (LLFP) can be found in transportation, problems of optimizing enterprise capital, the production development fund and the social, cultural and construction fund. Teterev [14] derived an optimality criteria for (LLFP) using the simplex type algorithm. Several authors studied the problem (LLFP) and its variants and have discussed their solution properties [2,4,6, 10,11,14].

In practical applications the data collected may not be precise, we would like to know the effect of data perturbation on the optimal solution. Hence, the study of sensitivity analysis is of great importance. In general, the main focus of sensitivity analysis is on simultaneous and independent perturbation of the parameters. Besides this, all the parameters are required to be analyzed at their independent levels of sensitivity. If one parameter is more sensitive than the others, the tolerance region characterized by treating all the parameters at equal levels of sensitivity would be too small for the less sensitive parameters. If the decision maker has the prior knowledge that some parameters can be given unlimited variations without affecting the original solution then we consider those parameters as 'nonfocal' and these 'nonfocal' parameters can be deleted from the analysis. Wang and Huang [15,16] proposed the concept of maximum volume in the tolerance region for the multiparametric sensitivity analysis of a single objective linear programming problem. Their theory allows the more sensitive parameters called as 'focal' to be investigated at their independent levels of sensitivity, simultaneously and independently. This approach is a significant improvement over the earlier approaches primarily because besides reducing the number of parameters in the final analysis, it also handles the perturbation parameters with greater flexibility by allowing them to be investigated at their independent levels of sensitivity. Singh et al. [13] extended the results of Wang and Huang [17] to discuss multiparametric sensitivity