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An algorithm for solving DAEs with mechanization ☆

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Abstract

In this paper, by using the theories and methods of mathematical analysis and computer algebra, a reliable algorithm of Padé approximation method for solving differential-algebraic equation systems was established, and a new `Maple` procedures `mainproc` was established, too. Ten examples are presented to illustrate the implementation of the program.

Keywords: Differential-algebraic equations; Algorithm; Power series; Padé series; Mechanization; `Maple`

1. Introduction

The rapid development of computer science and computer algebra system has a profound effect on the concept and the methods of mathematical researches [1,2].

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Differential equation is a deductive science and a branch of mathematics. At the same time, it is very important to remember that differential equation has strong roots in physical problems and that it derives much of its power and beauty from the variety of its applications [3]. As we know, differential equation is also an important tool of solving real-world problems. The differential equation has been applied to a wide class of deterministic and stochastic problems, linear and nonlinear, in physics, engineer, biology and chemical reaction and so on. But it is a difficult point in the field of solving differential equation or equations.

Differential-algebraic equation systems (DAEs) is an important kind of differential equations. Some numerical methods have been developed, using BDF and implicit Runge–Kutta method [4–8]. Besides, the numerical Padé method for DAEs has been developed in recent years [9–17].

However, the Padé method also requires a huge size of calculations. The objective of this paper is to establish a promising algorithm that can be easily programmed in `Maple`.

2. Basic methods

Let us first recall the basic principles of the Padé method for solving differential-algebraic equations [9–16]. Since these results are the key to the numerical manipulation for our problems, we give them in detail.

Consider the differential-algebraic equation:

$$F(x, y, y') = 0 \tag{2.1}$$

with initial values

$$y(0) = y_0, \quad y'(0) = y'_0,$$

where $F \in \mathbf{R}^n$ and $y \in \mathbf{R}^n$ are both vector functions for which we assumed sufficient differentiability and the initial values to be consistent, i.e.

$$F(0, y_0, y'_0) = 0. \tag{2.2}$$

According to the Taylor series method, the solution of (2.1) can be assumed that

$$y = y_0 + y'_0 x + C_1 x^2, \tag{2.3}$$

where C_1 is a vector function which is the same size as y_0 and y'_0. Substitute (2.3) into (2.1) and convert the elementary functions in (2.1) into series in $x = 0$, and neglect higher order terms, we will have the linear equation of C_1 in the form

$$A_1 C_1 = B_1, \tag{2.4}$$