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Fully developed flow through a porous channel bounded by flat plates

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Abstract

Plane, parallel and fully-developed flow through straight porous channels is considered in an attempt to study the effects of the porous matrix and the microscopic inertia on the velocity profiles, for different flow-driving mechanisms. By comparison, flows through free-space in the same configuration, as governed by the Navier–Stokes equations and subject to Poiseuille and Couette type entry profiles have become bench-mark problems in the study of flow equations that can be solved analytically by the method of reduction to ordinary differential equations. In this work, we therefore consider three types of Poiseuille–Couette combinations, together with the main models governing flow through porous media, and offer a comparison with the corresponding flow through free-space.

Keywords: Porous media; Flat plates; Flow models

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1. Introduction

Fluid flow through a porous medium is essentially a two-phase flow that is comprised of the flow of a fluid-phase and a matrix particle phase. However, it is usually assumed that the solid matrix is rigid, the particles are stationary, and hence the flow through the porous matrix is that of a single-phase fluid.

This type of flow has received considerable attention over the past century and a half due to its many applications and implications in the physical, biological, and applied sciences. Some of these applications include the movement of ground water, oil and gas through the porous earth layers. In addition, flow through porous media finds agricultural applications in irrigation processes and the movement of nutrients, fertilizers, and pollutants into plants (cf. [2] and the references therein).

The above, and a host of other applications, [1,2], emphasize the fundamental importance of seeking solutions to boundary and initial value problems in the field in order to accurately describe the flow patterns, the behaviour of the physical quantities in the medium, and the interactions of the flowing fluid with the porous matrix.

In the current work, we consider the plane, parallel and fully-developed flow through a straight porous channel in an attempt to study the effect of the porous matrix on the velocity profiles, for different flow-driving mechanisms.

By comparison, flows through free-space in the same configuration, as governed by the Navier–Stokes equations and subject to Poiseuille and Couette type entry profiles have become bench-mark problems, [4], in the study of flow equations that can be solved analytically by the method of reduction to ordinary differential equations. In order to document the effects of the porous matrix and the microscopic inertia on the flow profiles in the same configurations, we consider three types of Poiseuille–Couette combinations, together with the main models governing flow through porous media, and offer a comparison with the corresponding flow through free-space.

2. Governing equations

The flow of a viscous fluid is governed by the continuity and the Navier–Stokes equations which, when the flow is steady and the fluid is incompressible, take the form

$$\nabla \cdot \mathbf{v} = 0, \tag{1}$$

$$\rho(\mathbf{v} \cdot \nabla)\mathbf{v} = -\nabla p + \mu \nabla^2 \mathbf{v}. \tag{2}$$

In Eqs. (1) and (2), $\mathbf{v}$ is the velocity vector, ρ is the density, μ is the viscosity, p is the pressure, ∇ is the gradient operator and ∇^2 is the laplacian.