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Computational complexities and streamfunction coordinates

M.M. Awartani [a], R.A. Ford [b], M.H. Hamdan [b,*]

[a] *Department of Mathematics, Bir Zeit University West Bank, VIA Israel*
[b] *Department of Mathematical Sciences, University of New Brunswick, P.O. Box 5050, Saint John, New Brunswick, Canada E2L 4L5*

Abstract

In this work we consider some of the computational aspects of viscous fluid flow through two-dimensional curvilinear domains. Associated with the use of the von Mises transformation in mapping a physical plane into a computational plane are some complexities that arise in the main part due to the infinite nature of the Jacobian of the transformation. Some recommendations are made in this work to overcome some of the arising computational difficulties.

Keywords: Streamfunction coordinates; von Mises; Computational grid

1. Introduction

The last decade has witnessed a renewed interest in the well-known von Mises transformation, initiated by Barron [2], due to the direct adaptability of the

* Corresponding author.
E-mail address: mhhamdan@nb.sympatico.ca (M.H. Hamdan).

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transformation to the numerical treatment of single and multiphase fluid flow problems in complex geometries. The transformation has proved to be well-suited for the numerical simulation of two-dimensional, viscous and potential flow in curvilinear geometries possessing smooth boundaries, and major developments, advancements and extensions of the von Mises transformation have been achieved, as can be seen from the list of references herein (cf. [2,4,5,7,8,11]). Some of the advancements in this field have also been reported in a survey of the streamfunction-as-a-coordinate method in CFD, by Barron et al. [3].

A major break-through in the recent progress in the *computational von Mises* has been the introduction of a novel approach in the form of a double transformation, which has been applied to the study of two-phase flow in curved geometries [4,6]. The double transformation is capable of mapping a single physical domain where two phases coexist, onto a single computational domain. This idea was generalized to the study of n-phase fluid flow [7]. Superiority of this new approach is embedded in its utilization of the properties of the flowing phases (namely, using the streamlines as grid lines in the computational domain), thus providing a physically significant relationship between the coordinate transformation and the phases involved.

In the study of viscous fluid flow in curvilinear domains, the use of the von Mises coordinates received little attention, [8,14,15], and suffers from problems that are associated with the nature and definition of the Jacobian of transformation in terms of the tangential velocity component u, (namely, $J = 1/u$). This translates into the Jacobian becoming infinite at stagnation points; on no-slip boundaries; and when the streamlines of the flow are vertical, thus resulting in the non-uniqueness and invalidity of the transformation. The Jacobian changes sign at points and in regions where there is flow direction reversal, recirculation or viscous separation. Clearly, these cases endanger the one-to-one requirement of the transformation.

Although some of the above problems are not unique to viscous fluid flow, the use of a no-slip condition on a solid boundary adds a further complication to the implementation of the computational von Mises in the simulation of viscous fluid flow with separation and in the computation of the boundary vorticity. The nature of the transformation, together with the type of boundary conditions used, mandate the need for finite difference approximations with variable grid spacing and clustering of the coordinates near solid boundaries.

In this work, we analyze some of the problems associated with the use of the von Mises transformation in the study of viscous fluid flow in curvilinear domains. The computational complexity associated with the calculation of vorticity on the boundary; the choice of differencing scheme; and the choice of discretization and grid spacing are discussed. These influence both the global accuracy of the solution and the local accuracy of the derived vorticity boundary conditions. Various formulae are derived for approximating the vorticity on the boundary.