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Sensitivity and stability analysis in DEA

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Abstract

One of the topics of interests in data envelopment analysis (DEA) is sensitivity and stability analysis of specific decision making unit (DMU), which is under evaluation. In DEA efficient DMUs are of primary importance as they define the efficient frontier. In this paper, we develop a new sensitivity analysis approach for the CCR, BCC and additive models, when variations in the data are considered for a specific efficient DMU and the data for the remaining DMUs are assumed fixed.

Keywords: Data envelopment analysis; Efficiency; Sensitivity and stability analysis

1. Introduction

During the recent years, the issue of sensitivity and stability of data envelopment analysis (DEA) results has been extensively studied. The first DEA

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sensitivity analysis paper by Charnes et al. [2] examined change in a single output. This is followed by a series of sensitivity analysis articles by Charnes and Neralic [3] in which sufficient conditions preserving efficiency are determined. Another type of DEA sensitivity analysis is based on super-efficiency DEA approach in which a under evaluation DMU is not included in the reference set [1,7]. Charnes et al. [4,5] developed a super-efficiency DEA sensitivity analysis technique for the situation where simultaneous proportional change is assumed in all inputs and outputs for a specific DMU under consideration. This data variation condition is relaxed in Zhu [10] and Seiford and Zhu [8] to a situation where inputs or outputs can be changed individually and the largest stability region which encompasses that of Charnes et al. [4] is obtained. The DEA sensitivity analysis methods we have just reviewed are all developed for the situation where data variations are only applied to the under evaluation efficient DMU and the data for the remaining DMUs are assumed fixed. Obviously, this assumption may not be realistic, since possible data errors may occur in each DMU. Seiford and Zhu [9] generalize the technique in Zhu [10] and Seiford and Zhu [8] to the case where the efficiency of the under evaluation efficient DMU is deteriorating while the efficiencies of the other DMUs are improving.

The above sensitivity analysis methods are based on the increase of some or all inputs and the decrease of some or all the outputs. In this paper, we restrict our attention to the case where the increase of outputs and the decrease of inputs for DMU_p are not performed simultaneously, since the increase of outputs accompanied by the decrease of inputs cannot worsen the efficiency of DMU_p. In other word, the cases of (1) the increase of outputs and increase of inputs (2) the decrease of outputs and the increase of inputs (3) the decrease of outputs and the decrease of inputs can be performed. We wish to determine the largest region in which DMU_p remain efficient while the efficiency classification of the other DMUs are not changed. This region is called stability region. The stability region, just as have been shown in Fig. 1, is specified by using hyperplanes binding at DMU_p and the frontier of $\mathrm{DMU}_j (j \neq p)$ when the inputs and outputs of other DMUs are kept at their current levels.

The current article proceeds as follows. Section 2 presents some basic DEA models. Section 3 develops a sensitivity analysis method and provides a simple numerical example that illustrates the proposed method in this section. Finally, conclusions are given in Section 4.

2. Preliminaries

Consider a set of homogenous DMUs as $\mathrm{DMU}_j (j = 1, \ldots, n)$. Each DMU consumes m inputs to produce s outputs. Suppose that $X_j = (x_{1j}, \ldots, x_{mj})^{\mathrm{T}}$ and $Y_j = (y_{1j}, \ldots, y_{sj})^{\mathrm{T}}$ are the vectors of inputs and outputs values for DMU_j,