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A note on the componentwise perturbation bounds of matrix inverse and linear systems

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Abstract

In this note, we present sharp bounds for the componentwise perturbation of matrix inverse and linear systems, especially for nonsingular M-matrix. We also use matrix derivatives to deduce the matrix componentwise condition number and present some numerical tests to show our results.

Keywords: Matrix inverse; Nonsingular linear system; M-matrix; Componentwise; Perturbation

1. Introduction

It is well known that the classical condition number [7,8]

$$\kappa(A) = ||A^{-1}||||A||,$$

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relates the relative error of A^{-1}, measured in some matrix norm,

$$\frac{||(A+\Delta A)^{-1}-A^{-1}||}{||A^{-1}||} \leqslant \frac{\kappa(A)||\Delta A||/||A||}{1-\kappa(A)||\Delta A||/||A||}. \tag{1.1}$$

We also cite the known result for the componentwise perturbation of nonsingular linear system.

Lemma 1.1 [8]. *Let* $Ax = b$ *and* $(A + \Delta A)y = b + \Delta b$, *where* $|\Delta A| \leqslant \epsilon|A|$ *and* $|\Delta b| \leqslant \epsilon|b|$, *and assume that* $\epsilon|||A^{-1}||A||| < 1$, *where* $||.||$ *is a monotonic norm, then*

$$\frac{||x-y||}{||x||} \leqslant \frac{\epsilon}{1-\epsilon|||A^{-1}||A|||} \frac{|||A^{-1}||b|+|A^{-1}||A||x|||}{||x||}. \tag{1.2}$$

These results are normwise perturbation results and depend on the norm one shall use. Therefore it is reasonable to study componentwise perturbation bounds of matrix inverse and linear systems. It seems that Bauer [2] first obtained the componentwise perturbation bounds of A^{-1}, Rohn [11] derived the condition number of matrix inverse and nonsingular linear system. Xue and Jiang [15], Alfa et al. [1] studied the componentwise perturbations for nonsingular M-matrix and diagonally dominant M-matrix, respectively. Higham and Higham [6], Rump [12] considered the condition numbers with respect to structured perturbations in the input data for linear systems and for matrix inversion.

First we give some lemmas which we shall need in this paper.

An $n \times n$ matrix A is called an M-matrix if it can be expressed in the form

$$A = sI - B,$$

where B is nonnegative and $s \geqslant \rho(B)$, $\rho(B)$ the spectral radius of B. If $s > \rho(B)$, then A is nonsingular M-matrix.

Lemma 1.2 [4]. *Let* $X \in R^{n\times n}$. *If* $\rho(|X|) < 1$, *then* $I - |X|$ *is a nonsingular M-matrix and*

$$|(I+X)^{-1}| \leqslant (I-|X|)^{-1}.$$

To be convenient, let $Z^{n\times n}$ denote the matrix set $\{A|A \in R^{n\times n}, a_{ij} < 0, i \neq j\}$. We also let $M = \max_{ij}\{|a_{ij}|\}$ and $m = \min_{i,j}\{|a_{ij}|\}$. The following two lemmas are well known results. For details, we shall refer to [3].

Lemma 1.3. *Let* $A \in Z^{n\times n}$, *then the following statements are equivalent*:

(1) *A is a nonsingular M-matrix.*
(2) *A has all positive diagonal elements and there exists a positive diagonal matrix D such that AD is strictly diagonally dominant*, *that is*,