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An efficient convergent lattice algorithm for European Asian options

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Abstract

Financial options whose payoff depends critically on historical prices are called path-dependent options. Their prices are usually harder to calculate than options whose prices do not depend on past histories. Asian options are popular path-dependent derivatives, and it has been a long-standing problem to price them efficiently and accurately. No known exact pricing formulas are available to price them under the continuous-time Black–Scholes model. Although approximate pricing formulas exist, they lack accuracy guarantees. Asian options can be priced numerically on the lattice. A lattice divides the time to maturity into n equal-length time steps. The option price computed by the lattice converges to the option value under the Black–Scholes model as $n \to \infty$. Unfortunately, only subexponential-time algorithms are available if Asian options are to be priced on the

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lattice without approximations. Efficient approximation algorithms are available for the lattice. The fastest lattice algorithm published in the literature runs in $O(n^{3.5})$-time, whereas for the related PDE method, the fastest one runs in $O(n^3)$ time. This paper presents a new lattice algorithm that runs in $O(n^{2.5})$ time, the best in the literature for such methods. Our algorithm exploits the method of Lagrange multipliers to minimize the approximation error. Numerical results verify its accuracy and the excellent performance.

Keywords: Option pricing; Lattice; Path-dependent derivative; Asian option; Approximation algorithm; Lagrange multiplier

1. Introduction

Derivative securities are financial instruments whose values depend on some underlying assets. Such securities are essential to speculation and the management of financial risk. Options are financial derivatives that give their buyers the right but not the obligation to buy or sell the underlying assets for a contractual price called the exercise price. Take the typical stock option for example. Assume that an investor purchases a call option, which gives him the right to buy 100 shares of XYZ stock at \$10 per share 60 days from now. If the stock price ends above \$10 then, say \$25, then the buyer can realize a profit of $100 \times (25 - 10) = 1500$ dollars by exercising the option. If the stock price ends below \$10, the buyer simply gives up the option. The payoff of this call option is therefore $100 \times \max(S - 10, 0)$, where S is the stock price 60 days from now. Note that S is a random variable. This option is commonly called a vanilla option for its simplicity.

In practice, many varieties of complex options have been structured to meet specific financial goals. Take path-dependent options as an example. A path-dependent option is an option whose payoff depends nontrivially on the price history of the underlying asset, which we will assume to be stock for convenience. The payoff function may depend on the maximum stock price, the minimum stock price, or the average stock price, to mention just a few possibilities. It may also depend on whether the stock price ever hits a given target price, whether the stock price ever stays within two given target prices for a given length of time, and so on. The possibilities are clearly without limits.

How to assign a fair price to an option given a continuous-time stochastic process for the stock price has been investigated since as early as 1900 [1]. In 1973, Black and Scholes [2] settle the question for vanilla option pricing in a way that is considered intellectually satisfactory. Although an option must have a unique theoretical price, calculating that price may be computationally difficult if the payoff is complicated. For example, Chalasani et al. show that the general path-dependent option-pricing problem is #P-hard [3].