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Computations of shallow water equations with high-order central-upwind schemes on triangular meshes

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Abstract

New high-order central-upwind schemes on triangular meshes are proposed to approximate the solutions of shallow water equations. The nonuniform width of the different local Riemann fans is calculated more accurately, and the new central-upwind schemes are of simplicity, universality and robustness. At the same time, due to the central-upwind schemes are combined with new reconstructions based on adaptive least squares, the schemes have the almost non-oscillatory behavior. The numerical results show the desired accuracy, high-resolution, and robustness of our methods.

Keywords: Shallow water equations; Hyperbolic conservation laws; Balance laws; Central-upwind schemes; Triangular meshes

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1. Introduction

In this paper, the central-upwind schemes on triangular grids based on adaptive least squares are presented for solving 2-D shallow water equations

$$h_t + (hu)_x + (hv)_y = 0, \tag{1.1}$$

$$(hu)_t + \left(hu^2 + \frac{1}{2}gh^2\right)_x + (huv)_y = -ghB_x, \tag{1.2}$$

$$(hv)_t + (huv)_x + \left(hv^2 + \frac{1}{2}gh^2\right)_y = -ghB_y. \tag{1.3}$$

Here, $B(x)$ denotes the bottom elevation, h describes the fluid depth above the bottom, u and v represent the flow velocity in the x and y-directions respectively, and g is the gravitational constant.

The systems have steady states in which the flux gradients are nonzero but exactly balanced by source terms. In recent years, an amount of research was done in developing and implementing modern high-resolution finite difference methods for approximating solutions of the systems (1.1)–(1.3). These finite difference methods can be divided into two main categories, namely upwind schemes (see [2,12,1,10,13]) and central schemes (see [3]). The upwind schemes have to solve Riemann problems on the boundaries of each cell, which is interpreted as an upwinding procedure. However, a general scheme for the (exact or approximate) solution of the Riemann problems is not known, and the upwind approach may be rather complicated and costly, especially in multidimensional cases. On the other hand, by a straight-forward centered computation of the quantities involved, the central schemes avoid the Riemann solvers and characteristic decomposition of the Jacobians. Considering the local speeds of nonlinear wave propagation at the cells' boundaries, Kurganov et al. present central-upwind schemes in [9,5,7,6]. Since the nonuniform width of the overall local Riemann fans is calculated more accurately, the central-upwind schemes enjoy a much smaller numerical viscosity as well as the staggering between two neighboring sets of grids is avoided.

In this paper, we show the derivation of the new central-upwind schemes. Then we combine the new reconstructions based on adaptive least squares with the new central-upwind schemes. The central-upwind schemes in this paper are derived in general forms that are independent of the reconstruction step, as long as the reconstructed interpolant is sufficiently accurate and non-oscillatory. Finally, we test a second-order version of the proposed scheme on a large amount of examples.