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The tanh method for a reliable treatment of the K(n,n) and the KP(n,n) equations and its variants

Abdul-Majid Wazwaz

Department of Mathematics, Saint Xavier University, Chicago, IL 60655, USA

Abstract

In this work, the nonlinear K(n,n) and KP(n,n) equations and its variants are analytically studied. We show that these equations may exhibit compactons, solitons or periodic solutions by using the tanh method. The analysis reveals the change of the physical structure of the solutions as a result of the exponents and the coefficients of the derivatives.

Keywords: Compactons; Solitons; Nonlinear dispersion; Periodic solutions; The tanh method; K(n,n) equation, KP(n,n) equation

1. Introduction

The solitons exist as a result of the balance between the nonlinear convection term uu_x and the dispersion effect term u_{xxx} in the standard KdV equation

$$u_t + auu_x + bu_{xxx} = 0. \tag{1}$$

E-mail address: wazwaz@sxu.edu

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The integrable nonlinear KdV equation (1) with linear dispersion admits solitons: waves with infinite support. Solitons are defined as localized waves that propagate without change of its shape and velocity properties and stable against mutual collisions [1–5].

There are two well-known generalizations of the KdV equations, namely the integrable Kadomtsov–Petviashivilli (KP) equation [6], and the nonintegrable Zakharov–Kuznetsov (ZK) equation [7], given by

$$\{u_t + auu_x + u_{xxx}\}_x + ku_{yy} = 0, \tag{2}$$

and

$$u_t + auu_x + (\nabla^2 u)_x = 0, \tag{3}$$

respectively, where $\nabla^2 = \partial_x^2 + \partial_y^2 + \partial_z^2$ is the isotropic Laplacian [7–19].

The now well-known K(n,n) equation introduced in [8]

$$u_t + a(u^n)_x + (u^n)_{xxx} = 0, \quad n > 1, \tag{4}$$

where the delicate interaction between nonlinear convection with the genuine nonlinear dispersion generates solitary waves with exact compact support that are termed *compactons*. Compactons are defined as solitons with finite wavelengths or solitons free of exponential wings [9–19].

In modern physics, a suffix-on is used to indicate the particle property [1], for example phonon, photon, and soliton. For this reason, the solitary wave with compact support is called *compacton* to indicate that it has the property of a particle.

The stability analysis has shown that compacton solutions are stable, where the stability condition is satisfied for arbitrary values of the nonlinearity parameter. The stability of the compactons solutions was investigated by means of both linear stability and by Lyapunov stability criteria as well.

It is the objective of this work to further complement our previous studies in [10–12] on the K(n,n) equation. Our first interest in the present work being in implementing the tanh method [20,21] to stress its power in handling nonlinear equations so that one can apply it to models of various types of nonlinearity. The next interest is in the determination of exact travelling wave solutions with distinct physical structures to the K(n,n) equation given by

$$u_t + a(u^n)_x + b(u^n)_{xxx} = 0, \tag{5}$$

and to its related KP(n,n) equation in the (2+1)-dimensional, two spatial and one temporal variables, given by

$$\{u_t + a(u^n)_x + b(u^n)_{xxx}\}_x + ku_{yy} = 0. \tag{6}$$

Moreover, our analysis will extend the works of Rosenau [9] and Wazwaz [10–12] on the K(n+1,n+1) equation

$$u_t + a(u^{n+1})_x + b[u(u^n)_{xx}]_x = 0, \tag{7}$$