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New hysteresis operators with applications to counterterrorism

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Abstract

We define two models of hysteresis that generalize the Preisach model. The first model is deterministic, the second model is stochastic and it utilizes discontinuous transition probabilities that satisfy impulsive differential equations. For the first model we prove, among other things, a local version of the "wiping out" property; for the stochastic model, we give methods for the construction of solutions of impulsive differential equations that determine the discontinuous transition probabilities. We also present a game-theoretic problem utilizing a generalized hysteresis operator. These hysteresis operators are motivated by questions of modelling the dynamics of decision making processes of networks of loosely knit terrorist groups.

Keywords: Hysteresis; Impulsive Kolmogorov equation; Game theory

1. Introduction

This paper aims to provide a model of the decision-making processes of a loosely knit network of decision-making units. The underlying question is to model the behavior of a network of terrorist groups under conditions of "loose

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leadership''. We can take a clue from other problems that involve situations where preferences of several units affect a collective result. Such situations have been studied, in the context of Economics, in Refs. [10,17] and other papers. The conclusion is that collective responses to stimuli can, in certain situations, be modelled by Preisach hysteresis operators. In general, a situation involving several agents, each of which makes a binary decision in response to an applied stimulus, according to some rules, naturally leads to hysteresis, in the following way: a binary decision process in response to a scalar-valued stimulus can be expressed as a non-ideal relay; for the purposes of the present paper, it is convenient to label the two possible decisions as -1 and $+1$ and describe a non-ideal relay in terms of two subsets C_+, C_- of the real line $I\!R$, $C_- := (-\infty, \rho_1)$, $C_+ := (\rho_2, +\infty)$, $\rho_1 > \rho_2$, so that the decision changes from -1 to $+1$ when the stimulus exits from the set C_-, and switches from $+1$ to -1 when the stimulus exits from the set C_+; let us denote this relay by $R^{\rho_1\rho_2}$ the total effect, as a resultant of the decisions of the individual units, can then be represented as an operator of the form $H = \int\int_{\rho_1>\rho_2} w(\rho_1, \rho_2) R^{\rho_1\rho_2}\,\mathrm{d}\rho_1\,\mathrm{d}\rho_2$ where w is a weighting function; this means that, if u is an input signal, then

$$(Hu)(t) = \int\int_{\rho_1>\rho_2} w(\rho_1, \rho_2)(R^{\rho_1\rho_2}u)(t)\,\mathrm{d}\rho_1\,\mathrm{d}\rho_2.$$

For the case of multi-dimensional input signals, and multiple, rather than binary, decisions, we need an extension of this model, and a corresponding analysis of the properties of such generalized models. A first step in this direction is undertaken in the next section.

The importance of modeling and analyzing the decision-making processes of networks of terrorist groups is well understood [8,9,13,15,16,18]. Also, the works [11,12] utilize percolation studies to model the access of terrorists to targets via a social network; this last approach seems to us more suitable to homeland security issues rather than warfare in the terrorists' turf. Models of the decision processes and the behavior of terrorists can lead to techniques for data mining, in the sense of assimilating data into a theoretical model and identifying what kinds of data are useful and what kinds of missing data might lead to different analytical conclusions. In this paper, we aim at a mathematical analysis, and new mathematical constructs, based on our perception of what terrorists are, as gleaned from [1] and the factual information available in the news and in the quoted references. Our approach does not utilize existing models of terrorist decision making, but rather we look at the issue from a fresh point of view. Our main point of view is that terrorist groups have rational decision processes (rationality interpreted in a narrow sense), are connected through ''loose leadership'', and do not possess scientifically and computationally sophisticated systems of decision making. We model these characteristics by postulating that the decisions of each group, in response to external stimuli, can be represented