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On the representation of *k*-generalized Fibonacci and Lucas numbers

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Abstract

In this paper we give some determinantal and permanental representations of k-generalized Fibonacci and Lucas numbers. We obtain the Binet's formula for these sequences by using our representations.

Keywords: k-Generalized Fibonacci numbers; k-Generalized Lucas numbers; Hessenberg matrices

1. Introduction

Fibonacci numbers, Lucas numbers and their generalization have many interesting properties and applications to almost every fields of science and art. For the beauty and rich applications of these numbers and their relatives one can see Koshy's book and the nature.

Besides the usual Fibonacci and Lucas numbers many kinds of generalizations of these numbers have been presented in the literature (e.g. see [15,7,5,10,11,13]). One of these generalizations was given by Miles in 1960.

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For positive integer $k \geqslant 2$, the k-Fibonacci sequence $\{g_n^{(k)}\}$ is defined as:

$$g_1^{(k)} = g_2^{(k)} = \cdots = g_{k-2}^{(k)} = 0, \quad g_{k-1}^{(k)} = g_k^{(k)} = 1 \tag{1.1}$$

and for $n > k \geqslant 2$,

$$g_n^{(k)} = g_{n-1}^{(k)} + g_{n-2}^{(k)} + \cdots + g_{n-k}^{(k)},$$

where $g_n^{(k)}$ is called the nth k-generalized Fibonacci number. For example the 4-generalized Fibonacci sequence is

$$0, 0, 1, 1, 2, 4, 8, 15, 29, 56, 108, \ldots.$$

The k-generalized Lucas sequence $\{l_n^{(k)}\}$ is defined as

$$l_n^{(k)} = g_{n-1}^{(k)} + g_{n+k-1}^{(k)}$$

for $n \geqslant 1$, where $l_n^{(k)}$ is called nth k-generalized Lucas number. For example the 4-generalized Lucas sequence is

$$1, 2, 4, 9, 16, 31, 60, \ldots.$$

Silvester [18] obtained a matrix representation for usual Fibonacci sequence, and then Kalman [9] extended this representation for a generalization of Fibonacci sequence, that is the kth sequence of generalized order-k Fibonacci numbers. Then Er [5] represented all k sequences of the generalized order-k Fibonacci numbers by matrices. There are many kinds of representations of the generalized Fibonacci and Lucas numbers (see e.g. [17,12,14]) and a number of closed-form formula were obtained by using these matrix representations (see [21]).

In addition to these matrix representations for the usual Fibonacci and Lucas numbers were given in [3,6,2]. Authors in [4] obtained a complex factorization of usual Fibonacci and Lucas numbers by using representations given in [3].

Spickerman obtained the Binet's formula for tribonacci sequence [19], and for kth sequence of generalized order-k Fibonacci numbers [20]. Also, Lee [14] obtained the Binet's formula for k-generalized Fibonacci sequence in terms of Vandermonde determinants.

In this paper we give determinantal and permanental representations of k-generalized Fibonacci sequence and k-generalized Lucas sequence. Furthermore, we obtain the Binet's formula for these sequences by using our representations.

2. The determinantal representations

An $n \times n$ matrix $A_n = (a_{ij})$ is a lower Hessenberg matrix if $a_{ij} = 0$ when $j - i > 1$ i.e.,