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Extended Newton's method for a system of nonlinear equations by modified Adomian decomposition method

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Abstract

In this paper, we present some efficient numerical algorithms for solving a system of two nonlinear equations (with two variables) based on Newton's method. The modified Adomian decomposition method is applied to construct the numerical algorithms. Some numerical illustrations are given to show the efficiency of algorithms.

Keywords: Newton's method; Adomian decomposition method; System of nonlinear equations

1. Introduction

The decomposition method was first introduced by Adomian since the beginning of the 1980's for solving nonlinear functional equations [1–3]. Adomian gives the solution as an infinite series usually converging to an accurate solution. Abbaoui and Cherruault [4] applied the standard Adomian

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decomposition on simple iteration method to solve the equation $f(x) = 0$, where $f(x)$ is a nonlinear function, and proved the convergence of the series solution. Babolian et al. [5], modified the standard Adomian method which proposed in [4]. Abbasbandy [6] improved Newton–Raphson method to solve the nonlinear equation $f(x) = 0$ based on modified Adomian's method. Babolian et al. [11], applied the standard Adomian's method to solve a system of nonlinear equations.

It is the purpose of this paper to introduce an efficient extension of Newton's method by modified Adomian decomposition method. Some examples are tested, and the obtained results suggest that these techniques introduce a promising tool for solving system of nonlinear equations.

2. The Adomian decomposition method

Consider the system of nonlinear equations

$$\begin{cases} f(x,y) = 0, \\ g(x,y) = 0, \end{cases} \tag{1}$$

where (α, β) be a solution of it and f, g are continuously differentiable in an open convex set $D \subseteq R^2$ including (α, β). We suppose that the inverse of *Jacobian* matrix of (1) at (α, β) exists and is bounded, i.e., if

$$J(x,y) = \begin{bmatrix} f_x(x,y) & f_y(x,y) \\ g_x(x,y) & g_y(x,y) \end{bmatrix},$$

where f_x means that, the derivative of f with respect to x, and so on, then $J(\alpha, \beta)^{-1}$ exists with $\|J(\alpha, \beta)^{-1}\| \leqslant \eta$ for $\eta > 0$. Indeed, we assume that $J(x, y)$ to be *Lipschitz* continuous at the open neighborhood of radius $r > 0$ around (α, β). By using Taylor's expansion near (x, y)

$$\begin{cases} f(x-h, y-k) = f(x,y) - hf_x(x,y) - kf_y(x,y) + \mathrm{O}(h^2 + k^2 + hk), \\ g(x-h, y-k) = g(x,y) - hg_x(x,y) - kg_y(x,y) + \mathrm{O}(h^2 + k^2 + hk). \end{cases}$$

We are looking for a small h and k such as

$$\begin{cases} f(x-h, y-k) = 0 \approx f(x,y) - hf_x(x,y) - kf_y(x,y), \\ g(x-h, y-k) = 0 \approx g(x,y) - hg_x(x,y) - kg_y(x,y), \end{cases}$$

giving

$$\begin{bmatrix} h \\ k \end{bmatrix} = J(x,y)^{-1} \begin{bmatrix} f(x,y) \\ g(x,y) \end{bmatrix},$$