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Improved Muller method and Bisection method with global and asymptotic superlinear convergence of both point and interval for solving nonlinear equations

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Abstract

A new and improved version of Mullcr method and Bisection method with global and asymptotic superlinear convergence for finding a simple root x^* of a nonlinear equation $f(x) = 0$ in the interval $[a, b]$ is proposed in this paper. The new iteration procedure combines Muller method with Bisection method to generate simultaneously two sequences $\{x_n\}$ which goes to x^* and $\{[a_n, b_n]\}$ which encloses x^*. The global and superlinear convergence for the both sequences $\{x_n\}$ and $\{b_n - a_n\}$ are analyzed. The asymptotic efficiency index of the improved Muller method and Bisection method for the both sequences $\{x_n\}$ and $\{b_n - a_n\}$ proves to be 1.84 approximately on certain conditions, in the sense of Ostrowski. As a result, the new and improved version of Muller method and Bisection method preserve their respective nice property and remove their respective defect. The new version has been tested on a series of elementary functions. The numerical results show that the new version of Muller method and Bisection method

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proposed in this paper is more effective compared with the traditional version for solving nonlinear equations. For the computation of multiple zeros a effective strategy is discussed.

Keywords: Muller's method; Bisection method; Nonlinear equations; Root finding; Global convergence; Superlincar convergence; Convergence of interval diameter; Iteration method

1. Introduction

For finding a simple root of the nonlinear equation

$$f(x) = 0, \tag{1.1}$$

it is well known that Muller method [1] is a generalization of the Secant method which begins with two initial approximation x_0 and x_1 and determines the next approximation x_2 as the intersection of the x-axis with the line through $(x_0,f(x_0))$ and $(x_1,f(x_1))$. Muller's method employs three initial approximations x_0, x_1 and x_2 and determines the next approximation x_3 by considering the intersection of the *x-axis* with the parabola through $(x_i, f(x_i))$, $i = 0, 1, 2$.

The derivation of Muller method begins by considering the quadratic polynomial

$$P(x) = A(x - x_2)^2 + B(x - x_2) + C \tag{1.2}$$

that passes through the three points $(x_i,f(x_i))$, $i = 0, 1, 2$. The constants A, B, and C can be determined from the following conditions:

$$\begin{cases} f(x_0) = A(x_0 - x_2)^2 + B(x_0 - x_2) + C, \\ f(x_1) = A(x_1 - x_2)^2 + B(x_1 - x_2) + C, \\ f(x_2) = C \end{cases}$$

and to be

$$\begin{cases} A = \frac{(x_1-x_2)[f(x_0)-f(x_2)]-(x_0-x_2)[f(x_1)-f(x_2)]}{(x_0-x_2)(x_1-x_2)(x_0-x_1)}, \\ B = \frac{(x_0-x_2)^2[f(x_1)-f(x_2)]-(x_1-x_2)^2[f(x_1)-f(x_2)]}{(x_0-x_2)(x_1-x_2)(x_0-x_1)}, \\ C = f(x_2). \end{cases} \tag{1.3}$$

To determine x_3. the zero of $P(x)$ determined by (1.2), the quadratic formula is applied to $P(x)$. The formula is applied in the manner

$$x_3 - x_2 = \frac{-2C}{B \pm \sqrt{B^2 - 4AC}}, \tag{1.4}$$